

## GS-FrFT Algorithmic Method for Measurement of CD in Fibre Optics

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### Abstract

*The migration of present data network to the IoT interface, witty utilization of huge liable Single Mode Fiber rather than re-installing the whole network with Dispersion Shifted Fiber for their upgrading to ultra data speed Optical Fiber System based MPLS network for countering the main impediment of Chromatic Dispersion by estimating the real foe and compensating it at the receiver end and hence results the system more efficient with the existing Fiber network. The present scanning method of Fractional Fourier Transform (FrFT) to measure the Chromatic Dispersion (CD) encounters constraints of high latency time along with worst dispersion estimation error 250 picoseconds/nanometre which can be drastically improved with Golden Scan Search (GS) FrFT based algorithm method with 5 to 6 sec latency time and estimation error improved and obtained less than 80 picoseconds/nanometre.*

**Key-words:** GS FrFT Algorithm, OF Cable, Chromatic Dispersion (CD), Fractional Fourier Transform (FrFT), Single Mode (SM) Fiber, Fractional Convolution.

### 1. Introduction

The mobile communication players compete among themselves either upgrading or deploying their core network with holding the existing huge invested fiber back of Single mode fiber in the migration process from the present 4<sup>th</sup> Generation Mobile technology (4G) toward the 5<sup>th</sup> Generation Mobile technology (5G) network expansion. Ironically trade off with the propagation distance to ten folds with quadrupling the bits speed in the SM fiber. The main obstructing parameter for the present crisis is nothing but Chromatic dispersion. The data Scientist working hard in order to devise a practical solution for utilizes the present Single Mode (SM) fiber by estimating the Chromatic dispersion in the network and later can be easily compensated at the Rx end.

Fractional Fourier transform method to estimate the CD is the novel technique as this utilizes the technique to measure the total CD in the transmission media with adopting the scanning the optimum order with the MPLS or internet nature the transmission path and volatile nature of the optical transmission path. Hence the transmission lines path cannot be predicted. So we need a method to estimate the dispersion in the optical fiber independent of the transmission path. The estimation method should be free from the modulation technique the optical transmission path involve.

We have improved the method using the algorithm of GSS technique with drastically improved the plight time to obtain the optimum order and also measure accurate accumulated CD. [4].

Our research article paper organized in the following sections: The literature review article with the research work presented the recent methods related to the FrFT based CD estimation method explained in section II to find the research gaps in the existing models. The problem identification and solution has been stated in the next section. The proposed model is delineated in the section IV. The results and discussion of the article procedure of the present Scanning method and our algorithm based method is enumerated in the section V. Finally, the article conclusion part given in the section VI.

## **2. Literature Survey**

Huibin Zhou [23] [7] proposed a model FrFT based order scanning CD measurement technique in OF Cables and Systems. The reference method utilized finding of the optimum order with maximum likely hood of the availability based on the FrFT, Fractional convolution and power transformation methods. This method can be known as the scanning of order for getting the optimum order.

Aiying Yang, Xiang Liu [9] has proposed another model based FrFT method for measuring chromatic dispersion which is statistical method which used the Parseval's theorem to determine the optimum order upon making the FrFT on the received signal and calculating the power signal then making the chirp term to null hence finding the optimum order corresponding angle. But this method is hard to determine as involved the statistical methods.

Olcay Akay [21] [25] has given a solution to determine the optimum order based correlation method utilizing Fr Convolution method for demodulation of LFM Signals who had also utilized the Digital computation of FrFT algorithm proposed by Ozaktas and Kutay [22]. He devised two models

for the derivation of chirp parameter involved demodulation of the Frequency or Phase Modulation techniques. This can be utilized for getting the chirp parameter only in the range of -0.7 to 0.7, beyond this range the model cannot find the solution.

Adhemar Bultheel [18] has given a model for understanding of the Fr. FT then translated the formula helped to further develop an algorithm proposed by Ozaktas who taken the Ambiguity function as input to the FrFT for calculation of the total CD in the System for determining the optimum order and also have constraint with the optimum order range.

R. A. Soriano find a solution for determining the measurement of CD Optical Fiber System based on Polarization-diverse coherent demodulation and used tapered filter. Qi Sui and Alan Pak Tao also proposed a model for estimation of peak based on Power Transform backed with Fractional correlation in Optical Fiber Systems to determine the accumulated CD. This model developed the estimation method based on filter mechanism. This method works excellently when the transmission optical distance is fixed and CD is small but in the present scenario involves the MPLS nature and transmission path is large and unpredictable and also involves the resilient packet rings and optical systems for the best path search method.

### **3. Problem Identification**

Measurement of chromatic Dispersion for determining the LFM parameter based on measuring the chirp of CD in Time Frequency region using the existing statistical method, TD/frequency domain for calculating the optimum order. The chirp caused by CD is identified and the corresponding CD is depends on the precision of measurement well compromised with considering more number of orders with increasing the precision of scanning variable of fractional Fourier Transform. Hence requires involves more time constraints and also involves less degree of the precision of the optimum order. This model requires nearly 57 sec for determining the scanning of optimum order and accuracy of 0.8 % and of the accumulated CD and worst estimation error may be around 250 ps/nm.

The reference CD estimation method based on the Finding the optimum orders and scanning range. In contrast, we have proposed a model which satisfies the accuracy of estimation of CD improvement along with presenting the optimum order with the algorithm GSS (Golden Search Selection) Fr FT. In this method select a range of orders then determine the optimum order using FrFT corresponding to the order of the min max of the range. Then based on the magnitude of TF output iteration repeats and at last we can arrived the optimum order. Our work drastically reduces

the iteration time to nearly 5 sec for determining the scanning of optimum order and accuracy of 0.5 % and worst estimation error down to less than 80 ps/nm.

#### 4. Operation Principle

Fractional Fourier Transform can be a superset of all the transform, including Fourier transformation and that of STFT. Regarding Figure 1, which is of Time-Frequency plane and the original chirp signal can be transformed and fit in the TF plane and results in an acute clockwise rotation with the with time plane at an angle  $\alpha$  and with this instinct, we can calculate the optimum order  $a$  can be represented with  $2\alpha/\pi$  [2]. This Transformed signal can be designated as [1]

The time-frequency plane  $t$  and  $w$  and the newly transformed with angle  $\alpha$  will be  $u$  and  $v$  as shown in Fig 1 concerning the Weiner Domain. FrFT is the most generalized transform, and all others can be special cases and can say is the mother of all transforms. The anti clock wise movement with the transformed horizontal axis in accordance to the exclusive plane of transformed vertical plane can be expressed as the LFM rotation angle with  $\alpha$  which can be translated to the order which can be expressed the transformed chirp signal and the rotation angle can make equal to  $2\alpha/\pi$  [1].

The transformed signal can be expressed as and can be expressed as [1].

Fractional Fourier Transform dual is

$$F\alpha(u) = \int_{-\infty}^{\infty} f(t) K\alpha(t, u) dt$$

$$F(t) = \int_{-\infty}^{\infty} F(u) K\alpha(t, u) du \quad (1)$$

The main core of the equation  $K(t, u) =$

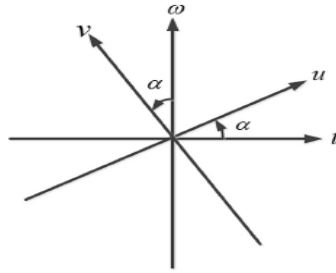
$$\left\{ \begin{array}{l} \sqrt{((1 - j\cot\alpha)/2\pi)} \exp(j \frac{(t^2+u^2)}{2} \cot \alpha - \frac{jt u}{\sin\alpha}) \\ \text{if } \alpha \text{ is not a multiple of } \pi \\ \delta(t - u) \\ \delta(t + u) \end{array} \right. \quad (2)$$

Here  $a = 2\alpha/\pi$

FrFT has multiple applications [1] in the field of Optical communication, Differential equations' solution, Radar communication, image signal processing and quantum mechanism.

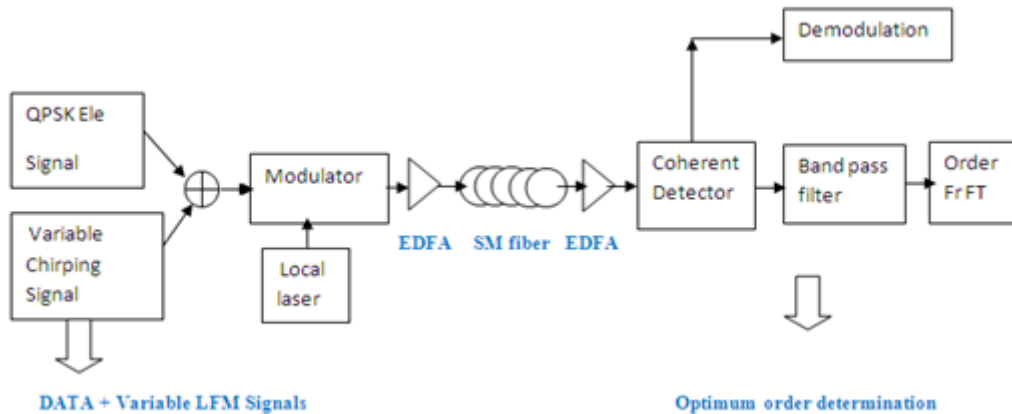
LFM signal transformed with FrFT will be best fitted the FT domain with the angle  $\alpha$  which is normal to the new  $f$  plane rotated with  $v$  plane as shown in Fig 1 and all the energy concentrated to the single condensed portion in the Weiner domain with the corresponding transform which can be expressed with the LFM parameter as shown in (1).

Figure 1 - Chirp signal angle with FT plan with an angle  $\alpha$



We have performed a practical with reference to the algorithm as shown in Figure 2 with utilizing the Optisystem 13.0 and obtained data has been analyzed with MATLAB 19. Input variable LFM signal has been modulated with respect to the DF Laser and amplified with EDFA amplifier and sent to the SM Fiber as shown below block diagram. The modulated signal at the receiver end again amplified with EDFA and stimulated to the coherent demodulator and output is filtered for the signal. The coherent output signal will be input to the proposed algorithm as shown in Figure 4.

Figure 2 - GS-FrFT Algorithm for CD calculation



The sequence of binary samples block with 32768 samples input to the FrFT transform let the input signal can be Fractional Transformed as shown in (3).

$$F_{\alpha} = X^{\alpha} f(t) \quad (3)$$

The transformed signal is convolved to get the optimum order corresponding to the maximum concentrated power can be resolved with (4) as FFT transform can be replaced with FrFT.

$$R_{\alpha}(a) = (F^{-1} \{ |F_{\alpha}(u)|^2 \}) (a) \quad (4)$$

for saving system resources with respect to Zhou model [7]. Scanning method can utilizes all the orders ranges [-4,4] and resolution also gives much difference in localization of the order for

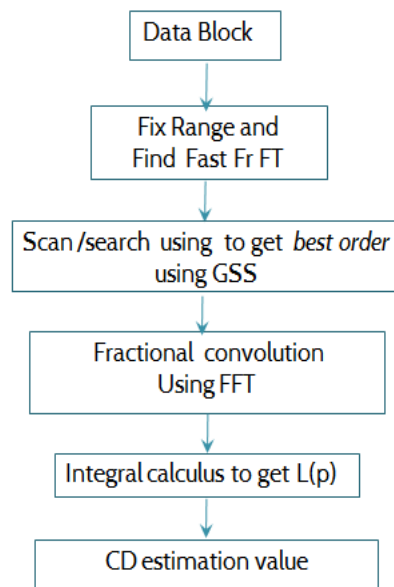
maximum  $L(a)$  as (5) and the scanning time can be reduced in our model with utilizing the Golden Search Section Algorithm as shown in the Flow chart.

$$L(a) = \int_{-\infty}^{\infty} |R_{\alpha}(a)|^2 da \quad (5)$$

With the inclusion of the algorithm in the Zhou model tremendously decreased the simulation as well as resolution of the order increased and hence the minimizes the error and very much reduces the simulation time.

Here Golden ration 5 is taken  $r$  value can be evaluated as 0.6180. As shown in GS flowchart as set the range  $[x, y]$  with min max orders and corresponding FrFT implemented and if  $F(a) < F(b)$ , then the existing range of orders is minimized with modified interval of  $(a ; y]$  otherwise,  $F(a) > F(b)$ , then the revised range becomes  $[x; b)$ . If  $F(a) = F(b)$  the new order can be between  $(a; b)$  because the orders  $a$  and  $b$  will be both sides of maxima the range is to be replaced as depicted in the flowchart revised order is picked hence the iteration can be reutilized.

Fig. 3 - Proposed GSS -FrFT Work Flow Diagram



So the new range is determined and the corresponding min and max fractional transform data is stored in the new  $F(a)$  and  $F(b)$  and if they are of equal then the optimum peak of the order is in between the order.

## Fr FT with GS Flowchart

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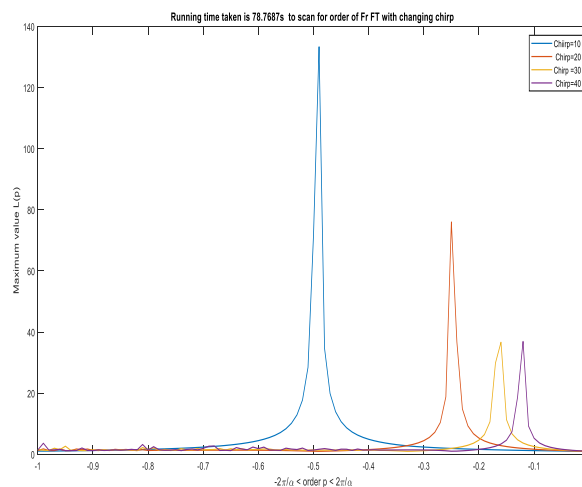
 $\alpha_1 \leftarrow \alpha_b - r (\alpha_b - \alpha_a),$ 
 $\alpha_2 \leftarrow \alpha_a + r (\alpha_b - \alpha_a),$ 
 $v_1 \leftarrow \max \{|f_{\alpha_1} \{c(t)\} (u)|\},$ 
 $v_2 \leftarrow \max \{|f_{\alpha_2} \{c(t)\} (u)|\},$ 
while  $|\alpha_b - \alpha_a| > \varepsilon$  do
  if  $v_1 < v_2$  then
     $\alpha_a \leftarrow \alpha_1$ 
     $\alpha_1 \leftarrow \alpha_2$ 
     $\alpha_2 \leftarrow \alpha_a + r (\alpha_b - \alpha_a)$ 
     $v_1 \leftarrow v_2$ 
     $v_2 \leftarrow \max \{|f_{\alpha_2} \{c(t)\} (u)|\}$ 
     $\alpha_{opt} \leftarrow \alpha_2$ 
  else
     $\alpha_b \leftarrow \alpha_2$ 
     $\alpha_2 \leftarrow \alpha_1$ 
     $\alpha_1 \leftarrow \alpha_b - r (\alpha_b - \alpha_a)$ 
     $v_2 \leftarrow v_1$ 
     $v_1 \leftarrow \max \{|f_{\alpha_1} \{c(t)\} (u)|\}$ 
     $\alpha_{opt} \leftarrow \alpha_1$ 
  end
end

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In this scenario the two orders has been stored in the memory and the process involves with the original scanning method but it is of very limited range of orders and the nature of precision sustained as it is.

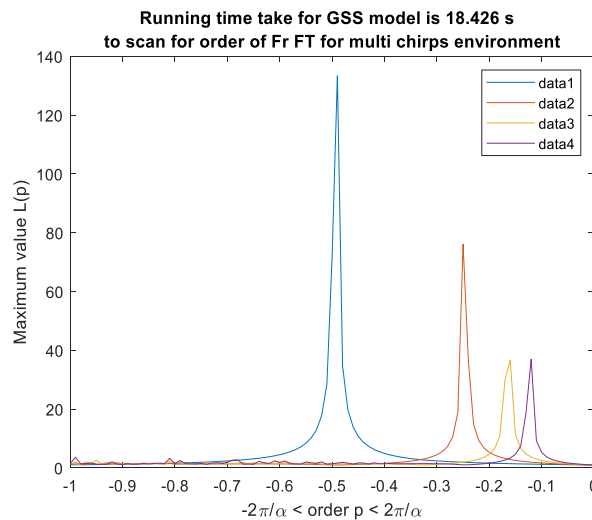
The range (a; b) can be resolved with  $a = y - r (y - x)$  and  $b = x + r (y - x)$  where  $r = (\sqrt{5} - 1) / 2$  nothing but GS ratio. The most important point i.e. every iteration results new orders range. The third case results then the iteration follow the old scenario. The b-a can be set as per our requirements to set to a desired limit until and unless the program continue to run.

Fig. 4 - Zhou model output for multi chirp signal



We have implemented one experiment with Nyquist rate as 8000 and  $k$  is the chirp /CD value and  $t = -1: Ts: 1-Ts$  with multi chirp signal  $exp j (2\pi f t + k t^2)$  Here  $k = k1 + k2 + k3 + k4$ .  $k1 = 0.12$ ,  $k2 = 1.4$ ,  $k3 = 0.3$  and  $k4 = 0.5$   $a$  is changed from 0 to 1 and performed Zhou model and results as shown in Figure 4 with time elapsed with 78.7687 sec.

Fig. 5 - GS –FrFT output for multi chirp signal



In this Figure we can show that maximum peaks obtained corresponding to the defined  $k$  values against the peak corresponding to the chirp parameter defined. So with this method we can detect the chirp in the LFM signal corresponding to the chirp parameter. With the optimum peak we can calculate the CD of the system using the equation (6). With calculating the CD and can be compensated it easily with the Digital Signal processing techniques.

The experiment repeated with GS flowchart method as shown in Figure 5 and time reduced to 18.426 sec. Here same algorithm repeated with the range of the data -1 and zero and corresponding  $F$  (a) and  $F$  (b) calculated and algorithm Figure 4 and flow chart followed while performing the iterations we have obtained the output shown in Figure 5.2. So this method improved with consuming very less system resources and hence it superior method.

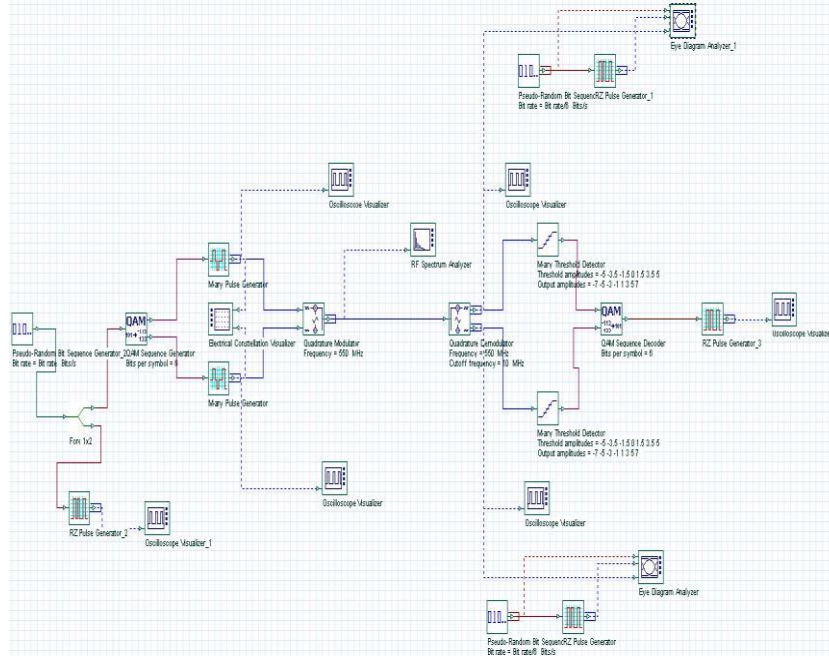
## 5. The GSS-FrFT CD Estimation Performance

We extended our experiment with optical Fiber Cable equipment's to examine the efficiency and modality of the measurement of the Chromatic Dispersion method. With reference to the Figure 6 a QPSK signal along with the CD embedded signal modulated with DF laser. The modulated signal is



amplified by EDF Amplifier and the modulated signal is propagated in the SM fiber. The fiber and amplifier are routed in a loop.

Figure 6 - Optisystem13 implementation for OF system



The loop parameter starts for 100km with 0.2 dB/km and the modulated amplified signals is looped from 1 to 10 times with dispersion of 16 ps/nm/km. The distorted signal is received at the receiver and is coherently modulated with the 37728 samples are given to MATLAB for the further analysis with performing the synthesis with (3) to (6) for determining of optimum order from Figure 7, it is evident that the maximum  $L(p)$  indicates the optimum peak and the CD corresponding to the order can be calculated (6) [6].

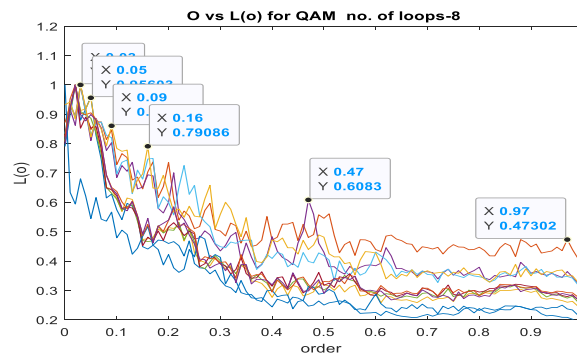
$$CD = \frac{\cot\left(\frac{\pi}{2} p_{optimum}\right)}{2 \pi S^2} \quad (6)$$

Here  $S = \text{SQRT}(n)$  and  $n$  represents the no. of samples of the Modulated wave. The results from the Optisystem 13.0 with various lengths of fiber from 100 km to 1000 km and corresponding data taken stored and moved to the MATLAB for the statistical analyses. The data is analyzed with the existing Zhou model and our GS-FrFT model for further analyses. Fig7.1 shows the results from reference model and has done with scanning method which is utilizing the scanning method to determine the optimum order to estimate the CD. Contrary to this Fig 7.2 shows the optimum orders

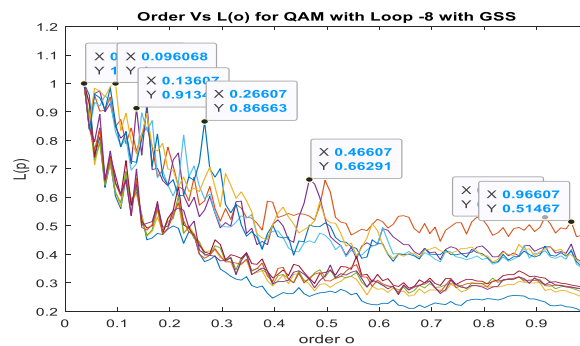
with much precision and hence the error in estimation decreases to the considerable level. The exact CD is calculated as well.

With reference to the Fig.5 the accumulated Chromatic dispersion varies with respect to corresponding fiber lengths. Here, we encounter the estimation errors to the corresponding distance for the SM optical cable length. The analyses with the GS FrFT results most proposing results in comparison with the Zhou model for estimation of accumulated Dispersion in the Single mode Optical Fiber. The orders resulted in our model has upto to 5 point resolution so we have obtained the more accurate CD in compared to the reference model as shown in Figure 7.

Figure 7(a) - Ghou Model order to L(p) (b) GSS model with order to L(p)



(a)



(b)

## 5.1 Comparing with Others Methods

On analyzing the Table 1 and Table 2 it can be inference that the most accumulated CD errors in the zhou model are 225 ps/nm and that of the GS-FrFT model is only below 80 ps/nm. Hence it can be proved that the latest analyses are the most suitable for the telecom players to adapt in their new

Table 1 Estimated CD Vs Actual CD of Zhou Model

S.No	Length of fiber (km)	Actual CD (ps/nm)	Estimated CD (ps/nm)	Diff.CD
1	100	1.6E-09	1.59E-09	1E-11
2	200	3.2E-09	3.22E-09	-2E-11
3	300	4.8E-09	4.77E-09	3E-11
4	400	6.4E-09	6.44E-09	-4E-11
5	500	8E-09	8.05E-09	-5E-11
6	600	9.6E-09	9.64E-09	-4E-11
7	700	1.12E-08	1.13E-08	-1E-10
8	800	1.28E-08	1.267E-08	1.3E-10
9	900	1.44E-08	1.444E-08	-4E-11
10	1000	1.6E-08	1.607E-08	-7E-11

Generation system to combat with the CD and hence can increase the range of communication. So the later is very attractive in the performance of CD estimation for bit-error ratio (BER) performance. Thus, this new approach no doubt can be adopted to operate well with different modulation formats along with various transmission distances

Our work with respect to the Zhou analyses is enumerated in table 1 to estimate the dispersion in the single mode fiber has that been summarized. Actual CD can be calculated with fiber length and dispersion in our calculations we have taken it as 16 ps/nm/km. Estimated CD can be calculated with Eq (6) corresponding to the optimum order obtained. Difference of the actual and estimated CD has been enumerated in the Table 1. This process has been repeated for every 100km and up to 1000km and results been posted.

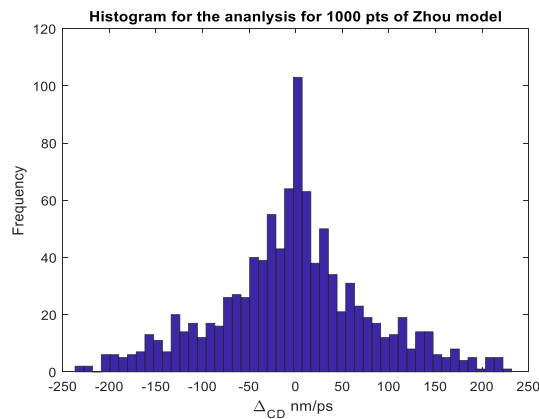
With reference to the Figure 6 data obtained in the optisystem 13.0 has been passes to the MATLAB using the plug-in tools.Data has been stored in the matrix tables has been processed using the GS algorithm and flowchart presented in Fig 4 and after completing the iteration process the results have been posted in Table 2 with stating the estimation CD values with the proposed model.

Table 2 - Estimated CD Vs Actual CD of Our GS FrFT

S.No	Length of fiber (km)	Actual CD (ps/nm)	Estimated CD (ps/nm)	Diff.CD
1	100	1.6E-09	1.59E-09	1E-11
2	200	3.2E-09	3.22E-09	-2E-11
3	300	4.8E-09	4.82E-09	-2E-11
4	400	6.4E-09	6.39E-09	1E-11
5	500	8E-09	8.04E-09	-4E-11
6	600	9.6E-09	9.63E-09	-3E-11
7	700	1.12E-08	1.119E-08	1E-11
8	800	1.28E-08	1.283E-08	-3E-11
9	900	1.44E-08	1.442E-08	-2E-11
10	1000	1.6E-08	1.596E-08	4E-11

In the Tables 1 and 2 given the actual CD and the resultant estimated CD has been presented. It is inevitable to know that the later has very less estimated error as the frequency of the errors distributed upto -225 ps/nm to +250 ps/nm as shown in Fig 8 H. Zhou model [8] and contrary to this our model as shown in the Fig 9 gives the worst error below 80 ps/nm.

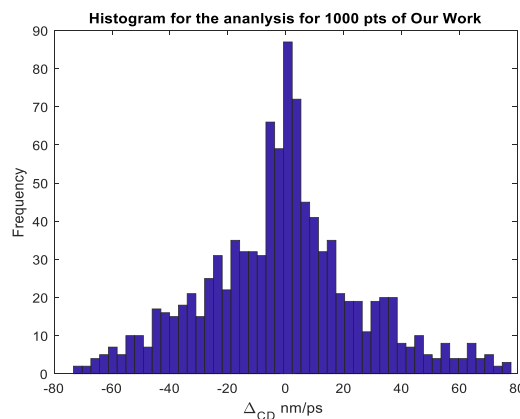
Fig. 8 - Histogram analyses for Zhou model



Now the we have taken 1km optical fiber with EDF amplifier and looped the process 1000 times and the output data in each iteration stored in the Matlab files and finally calculated the CD with the reference and proposed model and data has been analyzed and statistically comparing with Fig 8 to Fig 9 with histogram analysis of our model with zhou and posted the results of comparisons of estimation errors in Fig 9 and Fig 10.

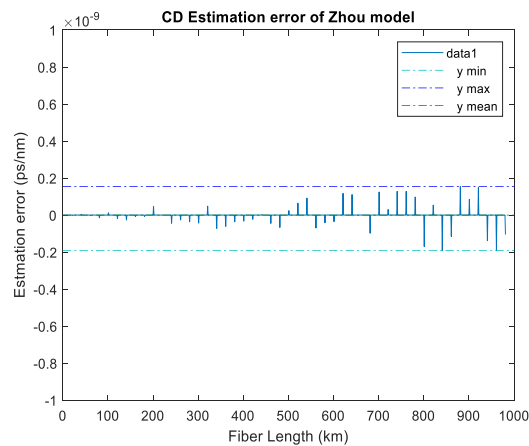
It is obvious that the most frequencies of the spikes in the histogram shown towards the zeros for our model as compared to the Zhou model [8], which is proposed and demonstrated in the year 2016 based on FrFT technology to measure CD.

Figure 9 - Histogram analyses Our GSS Work

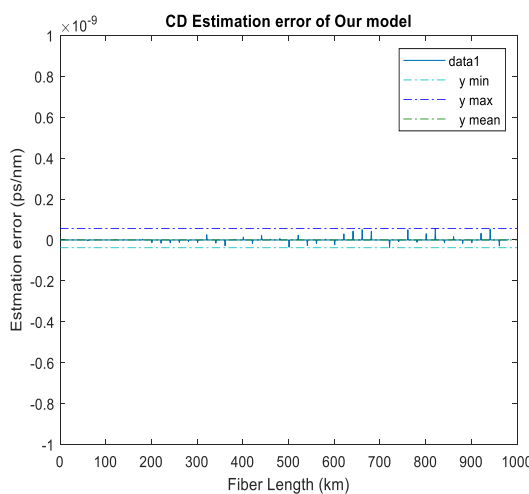


The latest proposed model is superior in the context of absolute error, standard deviation, worst case estimated error in both positive and negative direction as shown in the Fig 10 and Table 3. The proposed dispersion calculation method is very robust against LASER source ASE noise. 32786 samples against 2048 samples are used for CD measurement for a single carrier optical signal. The proposed model gives average.

Figure 10(a) - Error analyses for the Zhou model (b) - Error Analysis GS-FrFT Model



(a)



(b)

The dispersion analyses [7] [8] to estimated for chirping signal scanning technique have demerits with time constraint for scanning the order for the dynamic telecom network ie DWDM based resilient packet rings. This difficult has been addressed in the proposed model as depicted in the table 3.

## 6. Conclusion

The proposed model avoids the scanning of more and more orders with the proposed model with Golden Search Scanning Flow chart method which gives the optimum order with precision in determining the exact CD estimation with absolute error only of 20.25 ps/ nm and standard deviation nearly 3 times improved with the reference model.

Table 3 - Performance of the two estimation method

Method of CD Estimation	Absolute estimation error (ps/nm)	Standard Deviation (ps/nm)	Worst case estimation error (ps/nm)	Worst case estimation error (ps/nm)
Zhou Model	61.72	19.34	226.43	-220.79
GSS Model	20.25	6.34	76.44	-73.27

The Table 3 depicts the improvement of our proposed model with reference to the Zhou model. The worst case estimation error with the original error in both positive and negative directions as shown in Fig 10b and is below 80 ps/nm. The algorithm works independently for any type of modulation technique. The working model simulated with the OPTISYSTEM 13.0 and finding of optimum orders algorithm and statistical analyzed done with the GS algorithm. With reference to the Figure 10b maximum difference between the actual and estimated CD will be less than 80 nm.

## 7. Conflicts of Interest

The authors declare no conflict of interest.

## Author Contributions

The Research article's Literature review, procedure involved to estimation of CD, Software like MATLAB and Optisystem, Data extraction from Optisystem, Algorithm design and validation, Data analysis, investigation, writing—original draft preparation, writing—review and editing, visualization, have been done by 1st author. The supervision, and project guidance, has been done by 2nd author.

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