

Designing Mathematical Tasks for Primary School Students as A Means of Developing Combinatorial Abilities

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Abstract

The article has been devoted to the description of the study of the effectiveness of methodological techniques used in classes on the academic subject – "methods of teaching mathematics" with future primary school teachers. The proposed techniques allow activating the mutual connection of logical thinking and general awareness of students. This mutual relationship has been considered as a means to evaluate and develop combinatorial abilities in future primary school teachers. The assessment of the general cognitive abilities of students was carried out using the intelligence test (Eysenck, Raven). To assess the general awareness of students (the level of general cultural development, general humanitarian training, features of the microenvironment, as well as the level of general psychosocial development, etc.), the following methods were used: assessment of academic success; expert assessment of the level of general cultural development; assessment of the psychosocial development level. Original methodological techniques were used to stimulate students' interest in designing text-based mathematical tasks for primary school students at the formative stage of the study. The proposed methodological solutions have shown their effectiveness as a means of solving diagnostic tasks and developing combinatorial thinking in future teachers. The development of compositions of text mathematical tasks for primary school students is an effective tool for the development of students' combinatorial abilities. The proposed methodological solutions may be in demand in the training of future primary school teachers.

Key-words: Future Primary School Teachers, Methods of Teaching Mathematics, Compositions of Verbal Mathematical Tasks, Development of combinatorial Thinking, Cognitive and Non-cognitive Factors.

1. Introduction

The combination of cognitive factors and the degree of general awareness of students creates the basis for the development of their combinatorial thinking, which is a system of ways to search for various options, permutations, combinations, and placement of elements in those relations that are

determined by the conditions of the task and its purpose. Developed combinatorial thinking, combining the ability to think logically and simultaneously mobilize all their non-cognitive resources, is an important condition for the success of students in their future professional activities in the field of primary education. Concerning the relevance of the problem of the development of combinatorial thinking, researchers first of all pay attention to the close connection between the ability to solve tasks using combinatorial abilities, with the academic and general success of a person in life [R.A. Kutbiddinova, A.A. Eromasova and M.A. Romanova, 2016; D.J. Purpura and S.A. Schmitt, 2019; B. Rittle-Johnson, E.L. Zippert and K.L. Boice, 2019; V.I. Gliesburg, 2019; A.N. Kalinchenko and M.A. Romanova, 2020; V.M. Postavnev, I.V. Postavneva, A.M. Dvoinin and M.A. Romanova, 2020, etc.].

Combinatorial abilities in the conditions of everyday pedagogical activity are required for a modern teacher to solve a variety of professional tasks, both theoretical and practical. Important evidence of the development of the combinatorial abilities of the individual is the productivity of thinking when solving tasks of the divergent type. It creates opportunities to achieve an original result by analyzing, comparing, and combining a large number of options [T.N. Tikhomirova, 2015; A.I. Savenkov and M.A. Romanova, 2020, etc.]. Therewith, the productivity of thinking is only one of the conditions for developed combinatorial thinking. The ability of the individual to produce new, non-banal, in the full sense of the word, original ideas is fundamentally important here. No less important in the process of combinatorial thinking is the ability to use different decision strategies. In addition, it is important to be able to bring these original ideas, obtained as a result of the development of a large number of original ideas using fundamentally different strategies, to a product suitable for use [E.A. Valueva and D.V. Ushakov, 2017; E.I. Shcheblanova, 2017; A.I. Savenkov, S.I. Karpova and E.I. Sukhova, 2018; B. Vucichevich and N.B. Shumakova, 2020, etc.]. In our case, such a product is mathematical, text-based tasks for primary school students, which allow children to be keen on mathematics, to demonstrate to them that mathematics is not a boring science, it can be extremely interesting.

2. Theoretical Basis

Philosophical and psychological research on the inseparable connection of the cognitive and psychosocial spheres of personality has a long tradition in Russian psychological science. The problem of the inseparable connection between intellect and effect was first identified in Russian psychology by L.S. Vygotsky and developed by his students and followers (M.Ya. Basov, V.V.

Davydov, V.P. Zinchenko, A.M. Matyushkin, V.V. Rubtsov, and others). Modern psychologists, sharing M. Polanyi's idea of personal knowledge, emphasize that the properties of objects that are outside our field of vision and are not consciously perceived, inaccessible to conscious control, do not disappear without a trace, they become the basis of intuitive experience that appears regardless of the desire of the individual, who subsequently will necessarily manifest itself in thinking and behavior [M. Polanyi, 1985; D. Berry and D. Broadbent, 1995; B.M. Velichkovsky, G.G. Knyazev, E.A. Valueva and D.V. Ushakov, 2019; and etc.].

However, studies devoted to the research of the factors that determine the cognitive productivity of an individual in the process of solving divergent tasks indicate well-known discrepancies among specialists. Some researchers argue that the main role is played by cognitive factors (R. Sternberg; E. L. Grigorenko, 2019; T. N. Tikhomirova, Yu.V. Kuzmina and S.B. Malykh, 2020, etc.). Other researchers believe that non-cognitive factors are the main condition for success in these processes [S. Agnoli and others 2012; A.I. Savenkov and M.A. Romanova, 2009; J. Baptista and others, 2016; A. I. Savenkov, S. I. Karpova and E.I. Sukhova, 2018; T. N. Banshchikova, M. P. Sokolovskii and V.I. Morosanova, 2020; and others], among which are motivation, self-confidence, the style of attribution of successes and failures, etc.

Our research was based on the idea that a combination of two characteristics is of particular importance for the development of combinatorial abilities in future teachers: logical, consistent, unidirectional thinking, which develops in training, as a rule, at a conscious level and is the basis for the acquisition of declarative knowledge, and implicit, hidden in the depths of the subconscious, knowledge acquired in the course of general psychosocial development and the development of students in a variety of academic disciplines. Therefore, the main factors for the development of students' combinatorial thinking, along with cognitive ones, should be considered as the factors of students' general awareness: general humanitarian training, the breadth of interests and the degree of awareness in various fields of knowledge, the degree of success in mastering subjects of general scientific, general professional and special training.

3. Methods

The main participants in our study conducted in 2020 were undergraduate and graduate students of the Moscow City Pedagogical University, studying in the profiles of "primary education", training areas: "pedagogical education" and "psychological and pedagogical education". The total number of future primary school teachers who participated in our work – 132 people who worked

under the guidance of three teachers of the training module – methods of teaching mathematics in primary school.

The assessment of the development level of students' abilities to logical, consistent, unidirectional thinking was carried out based on computer testing. J. Raven's test (computer version) was used for this purpose. The level of general intellectual development was determined by standard tests of intelligence, which allow obtaining the coefficients of the intellectual development of the subjects in quantitative terms (G. Eysenck's Intelligence Test).

Two groups of factors of general awareness of future primary school teachers – general humanitarian training and breadth of interests – were evaluated by experts in the course of interviews and observations. The average indicator for each parameter was calculated for each student to increase the degree of objectivity of the assessment given by experts. The third group of parameters – "the degree of success in the development of academic subjects" – was determined by the marks of students received during the development of the main educational programs.

Based on the theoretical provisions outlined above, we have adjusted the programs of the academic disciplines of the training modules: "Methods of teaching mathematics in primary school" and "Professional and personal development of the future teacher using information technologies". The structural elements of methodological support for the classroom and independent work of students were revised and clarified, and the technological maps of the studied disciplines were revised ("Methodological foundations of teaching mathematics", "Methods of teaching tasks solving in primary school").

We used methodological tasks that require independence of thinking, originality, ingenuity, and a high degree of novelty along with the classical methods of forming teachers' methodological skills related to teaching mathematics to younger students. As it turned out, the process of developing the future teacher's combinatorial skills, the ability to develop original plots of text mathematical tasks and solve these tasks in different ways, to think creatively is much more difficult than the process of mastering the main content of the initial course of mathematics and developing their style of theoretical thinking. It was possible to overcome the problem by setting psychological, pedagogical, and methodological tasks that require the direct application of such a style of thinking.

The obtained results were processed by statistical analysis methods. To analyze the relationship and interaction between the scales, J. Raven's test (computer version), G. Eysenck's intelligence test, correlation, and variance analysis were used.

4. Results

The greatest effect was achieved by tasks related to the development of compositions of mathematical tasks. Students, having mobilized their knowledge, constructed original plots of typical mathematical tasks; texts of tasks of increased complexity, as well as complex mathematical tasks in the PISSA format. These tasks allowed stimulating their abilities to divergent thinking allowed developing combinatorial abilities through competencies in the fields of algebra, geometry, mathematical analysis, and mathematical statistics, as well as general humanitarian training and awareness of the processes and phenomena of modern life.

We will illustrate the results of the experimental work with examples of the original texts of tasks that were constructed by students during the training.

Attention should be paid to texts based on works that are included in the circle of classroom and extracurricular reading or cartoons and help develop the reader's interest in younger students. As well as informative texts that expand horizons, with elements of storytelling. Here are some examples of such texts.

"Baron Munchausen told 22 incredible stories about the first trip to the moon, 17 stories about how he saved his bees from bears and 32 stories about a fight with a bear. How many stories did Baron Munchausen tell in total? " (Based on the Russian animated film "The Adventures of Baron Munchausen" by the Soyuzmultfilm film studio).

The Frog-Traveler Monument in Tomsk (Russia) is the smallest monument in the world. The height of this tiny bronze frog is 66 mm smaller than the statue of Chizhik-Pyzhik in St. Petersburg (Russia). What is the height of the smallest monument in the world, if the height of the "Chizhik-Pyzhik" is 11 cm?

Robot animals are used for filming wildlife films. It took 24 moving parts to create a "spy" of a wild dog, and it took 6 more only for the head of an orangutan. How many moving parts are there in the head of an orangutan robot?

Two jockeys at the races agreed that the winner will be the one whose horse comes to the finish line last. None of them dared to move after the start, for fear of losing. What advice would you give to these jockeys so that they can start racing following the agreement?

The tasks of local lore content related to the history of the native land are interesting. Students constructed sets of tasks connected by one thematic line. An example is a series of tasks of one of the students who took part in the study, Victoria M.

"Korolev is a small city in the Moscow region, but it is called the space capital of Russia. It was founded on December 26, 1938, and was called Kaliningrad. Since July 8, 1996, the city has been named after the scholar and designer in the field of rocket science and cosmonautics, Academician S.P. Korolev. How many years has the city been named Kaliningrad?"

"Sergei Pavlovich Korolev is a Soviet scholar and designer of rocket and space systems. He was born on January 12, 1907. He developed his first aircraft at the age of 17 – a non-motorized aircraft. He created the first spacecraft, Vostok-1, thirty-seven years later, on which the first flight into space was made. In what year was the first space flight made?"

"A monument has been unveiled in the city of Korolev in honor of the launch of the first Earth-satellite vehicle. Many call it the symbol of the city. The height of the monument is 24 m 48 cm: the granite pedestal of the monument is 14 m, the steel spire piercing the sphere is 10 m, the rest is a model of the satellite. What is the length of the satellite mock-up diameter? "

A whole series of text tasks was created to organize and conduct the All-Russian online quiz for primary school students "Regions of Russia". We present a text about the nature reserves of the Moscow region as an illustration.

"According to the geographical dictionary" All Moscow Region", the National Park "Zavidovo" is a nature reserve located in two regions, Tver and Moscow region. Its total area is 125 hectares*. Of these, 68 hectares are located in the Tver region. What is the area of a part of the reserve in the Moscow region (Russia)?"

A significant indicator of the combinatorial style of thinking is that the development of the text itself and the methods of solving the resulting tasks is carried out based on the multiplicity and variety of subjects for composing their compositions, on the one hand, and the limitations caused by the content of the initial course of mathematics, on the other. Therefore, this activity is directly related to the manifestation of a combinatorial style of thinking, in which the search for solutions is not random and "insight" occurs not only based on intuition.

5. Discussion

The empirical part of the study aimed to clarify the relationship and mutual influence of combinatorial abilities, which are manifested in the development of original compositions of verbal mathematical tasks for primary school students and the general awareness of the student, his/her competence in different spheres of life and activity.

Correlation and variance analysis was used to analyze the relationship and mutual influence between the scales (see Table 1).

Table 1. Empirical values of correlation analysis

	General humanitarian training	Combinatorial abilities	Psychosocial development	Horizon
Ability to think logically (IQ test, Raven, Amthauer)	0.842***	0.922***	0.839***	0.872***
General humanitarian training	—	0.704***	0.799***	0.823***
Breadth of interests	—	—	—	—
Success in mastering academic subjects	—	—	—	—
Combinatorial abilities	—	—	0.734***	0.769***

The analysis of the results allows formulating several conclusions that reflect the identified patterns. Thus, we have recorded that the growth of indicators on the "General humanitarian training" scale determines the growth of indicators on the "Combinatorial abilities" scale. The obtained value of $r=0.704$ ($p<0.001$) indicates the presence of significant strong positive relationships between the "General humanitarian training" and "Combinatorial abilities" scales.

The increase in the indicators on the "Logical thinking ability" scale also provides a tendency to increase the indicators on the "Combinatorial ability" scale. The correlation coefficient indicates the presence of significant strong positive relationships between the "Combinatorial abilities" scale and the "Logical thinking ability" factor ($r=0.922$; $p<0.001$).

There are significant strong positive relationships between the "Combinatorial abilities" and "Psychosocial development" scales ($r=0.734$; $p<0.001$). Respondents who have the severity of indicators on the "Psychosocial development" scale have higher indicators on the "Combinatorial abilities" scale.

Significant strong positive relationships were found between the "Combinatorial abilities" scale and the "Horizon" scale ($r=0.769$; $p<0.001$). High scores on the "Horizon" scale correlate with similar scores on the "Combinatorial Abilities" scale.

The conducted analysis of the obtained data allows stating that non-cognitive factors directly affect the development of combinatorial abilities of future teachers. We used Levene's test to confirm the conclusion and found a statistically significant influence of the selected non-cognitive factors.

The use of one-factor analysis of variance was sufficient to assess the impact of general humanitarian training on combinatorial abilities since no significant statistical differences between the variances were found ($F=2.957$; $p>0.05$). We see that the "General humanitarian training" variable affects the value on the "Combinatorial abilities" scale and this effect is statistically significant ($F=32,294$; $p<0.001$). The highest value is observed at 5 points (7.1 ± 1.3), the lowest values at 2 points (44.6 ± 4.0).

Differences between variances (Levene's test) are statistically significant, so we performed a one-factor analysis of variance taking into account Welch's t-test. The influence of the "Horizon" variable (and these values are statistically significant) on the values on the "Combinatorial abilities" scale ($F=7,126$; $p<0.001$) was established. The highest value is observed at a high level (57.5 ± 0.5), the lowest values at a low level (46.5 ± 4.9).

There was also a statistically significant influence of psychosocial development on the values of the "Combinatorial abilities" scale ($F=7,126$; $p<0.001$). The highest value is observed at a high level (57.5 ± 0.5), the lowest values at a low level (46.5 ± 4.9).

Due to the absence of statistically significant differences between the variances according to Levene's test ($F=3,117$; $p>0.05$), the degree of influence of the level of development of logical thinking abilities on combinatorial abilities was carried out using single-factor analysis of variance. Here, we found a statistically significant influence of the value "Theoretical foundations of the initial course of mathematics" on the values according to the "Combinatorial abilities" scale ($F=30,806$; $p<0.001$). The highest value is observed at the level of 5 points (56.0 ± 1.6), the lowest values at the level of 3 points (46.5 ± 4.9).

Finally, to determine the overall level of the intellectual development of students, we used two subscales of the G. Eysenck intelligence test, which characterize general intelligence. We concluded that if future primary school math teachers have reserves of intellectual development, then the intelligence coefficients that characterize it tend to increase.

6. Conclusion

The development of combinatorial thinking in future teachers is understood by us as a combination of developed unidirectional, convergent thinking and several factors that characterize the general awareness of students, determined by evaluating the coefficients of general intellectual development, general humanitarian training, the breadth of interests and the degree of success in mastering academic subjects. The study showed the effectiveness of the proposed methodological

solution to the problem of developing combinatorial abilities in students by working out in the classroom a methodology for teaching mathematics, plots of verbal, mathematical tasks for primary school children, of various levels of complexity using material from different spheres of life and human activity. The development of compositions of mathematical tasks for primary school students requires future teachers to mobilize their cognitive abilities and activate general competencies in different areas of life and activity and is an effective means of developing their combinatorial abilities.

A significant indicator of the combinatorial style of thinking is that the development of the text itself and the methods of solving the resulting tasks is carried out based on the multiplicity and variety of subjects for composing their compositions, on the one hand, and the limitations caused by the content of the initial course of mathematics, on the other. Therefore, this activity is directly related to the manifestation of a combinatorial style of thinking, in which the search for solutions is not random and "insight" occurs not only based on intuition.

The resource of constructing verbal mathematical tasks for younger students should be actively used in the practice of professional education of future mathematics teachers in primary school. Tasks of this kind activate the students' combinatorial thinking, which manifests itself directly in the professional sphere.

Acknowledgments. The organizers of the research thank all the students who took part in the research and express their gratitude for their participation in the First International Student Competition of Local Lore Mathematical Tasks for Younger Schoolchildren dedicated to the 80th anniversary of Professor Tolegen Karazhanovich Ospanov (Almaty), at which the research participants became laureates of the first, second and third-degree.

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