

Possibilities of Using the "Analogy" Method in Teaching Mathematics

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Abstract

This article is devoted to the possibility of using the method of inference "Analogy" in the teaching of mathematics, in which the method of traditional inference, analogy method, incomplete analogy, analogy with mathematical accuracy, strict and indeterminate analogy, the effectiveness of teaching through the appropriate use of mathematics. upgrade options are disclosed.

Key-words. Traduction, Analogy, Traductive Inference, Quality of Education, Learning Efficiency, Incomplete Analogy, Analogy with Mathematical Accuracy, Strict Analogy, Indeterminate Analogy.

1. Introduction

Ensuring the quality and effectiveness of education - not only ensures the acquisition of knowledge by students, but also increases their interest in the subject. In order to achieve the above in the process of teaching mathematics, it is important to use the "methods of inference", especially the method of "analogy". However, the use of the method of "Analogy" in the study of mathematics in general secondary education has not been sufficiently studied. Therefore, in this article we want to focus on the possibilities of using the method of "Analogy" and its importance.

2. The Main Results and Findings

Definition. Drawing a general conclusion from one or more general concepts about one or more other concepts with such a degree of accuracy is called a traditional conclusion.

The word "traduktsiya" is derived from the Latin word "traduktio", which in Uzbek means "displacement", "relocation"[3,4,5].

For example: Suppose, $\forall a, b, c \in R$ for: $a > b$ (Concept 1) and $b > c$ (Concept 2) get In that case, $a > c$ (new concept) it turns out that. Indeed, $(\forall a, b, c \in R)(a > b \wedge b > c \Rightarrow a > c)$ will be.

The meaning of translation as a method of inference is as follows: To draw a conclusion about the similarity of two objects in another relationship based on the similarity of two objects in one relationship.

The main type of translational inference is analogy, which is derived from the Greek word "analogia" and means "similar" in Uzbek[3].

The conclusion by analogy is made in the following order:

Confirmation 1. A object a, b, c, x, \dots properties.

Confirmation 2. B object a, b, c, \dots properties.

Conclusion: B object x as well as properties.

So, to draw a conclusion with the help of analogy is to transfer this knowledge to another model of an object that has not been fully analyzed, based on the knowledge gained as a result of studying the model of one object [4,5].

For example: A right rectangle in a plane is an analogous quality to a right-angled parallelepiped in space, because the relations defined for a right rectangle are similar to the relations between the sides of a parallelepiped, i.e:

the opposite sides of a right rectangle are mutually parallel and equal;

the two sides of a right rectangle are mutually perpendicular;

the opposite sides of a right-angled parallelepiped are mutually parallel and equal;

the sides of a right-angled parallelepiped are perpendicular to its bases.

From the above it can be seen that the properties of a rectangle and a right-angled parallelepiped have the following similarity relations: "parallelism", "equality" and "perpendicularity"[3,4].

Thus, the relationship between the two objects (similarity) analogiyaga example of the denial. However, it should be noted that any similarity is not an analogy, so the analogy is at the following levels [3,4,5,6]: 1. Incomplete analogy. 2. An analogy with mathematical accuracy.

In turn, the analogy is divided into the following types:

1. Based on the similarity of some properties of two or more objects, a general conclusion about the similarity of their other properties is an analogy, and such a conclusion is usually called a simple analogy.
2. An analogy that leads to causal similarity from the similarity of objects. This in turn is divided into the following types [2,4,5,6]:

a) An analogy in which the signs of the objects being compared are clearly visible to each other. This is commonly referred to as a strict analogy.

b) an analogy in which the similarity of the signs of the objects being compared is not clearly visible. This is often referred to as a rigid analogy.

The conclusion is as a method of using the analogy in some cases can be straight, which is closer to conclusion. In this case, the result of experience through the analysis required to check.

The main type of analogy that leads to the correct conclusion is isomorphism.

When declaring the isomorphism of a system of two or more objects, it is possible to move an arbitrary character belonging to one of these systems to another. In this case, it is possible to draw a general conclusion by examining any one system of objects without examining the objects that are isomorphic to it [4,7,8].

For example. The geometric figures studied in the geometry course and their properties correspond to the analytical relations applied to numerical objects.

Let us now consider examples of analogy:

1. The square of the length of a diagonal of a right rectangle is equal to the sum of the squares of the lengths of its two sides: $d^2 = a^2 + b^2$.
2. With respect to this assertion, "the square of the length of the diagonal of a right-angled parallelepiped is equal to the sum of the squares of its three dimensions." $d^2 = a^2 + b^2 + c^2$ check analogy.
3. The face of a right rectangle $S = a \cdot b$ is calculated using the formula "- that statement "The size of a rectangular parallelepiped $V = a \cdot b \cdot c$ is calculated using the formula "will be the confirmation that analogy.
4. $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$ depending on the formula, $\sqrt{-a} \cdot \sqrt{-b} = \sqrt{ab}$ Will he writes?

Answer: Yes.

5. $a + b = c$ from $a^2 + b^2 = c^2$ and $a^3 + b^3 = c^3$ will be. This conclusion is correct?

Answer: No.

$$6. \quad (a+b)^1 = a+b; \quad (a+b)^2 = a^2 + 2ab + b^2; \quad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3;$$

$$(a+b)^4 = a^4 + 4a^3b + \frac{4 \cdot 3}{2} \cdot a^2b^2 + 4ab^3 + b^4 \quad \dots$$

Answer: Yes.

Scientific research and many years of pedagogical experience show that analogy plays an important role in the development of students' search skills, and the possibility of using analogy is high in the following cases [3]:

- in comprehensively understanding the definition of a concept;
- in consciously mastering the properties of the figure;
- in determining the geometric position of points;
- in proving theorems (problems);
- in search of ways to solve (prove) the problem (theorem), etc.

By analogy, let us focus on the possibilities listed above. To do this, consider the following definitions learned in the Planimetry course:

- I. The plane is a point from the same underlying geometry (points) to the role of the dots is called the circle.
- II. The vatar passing through the center of the circle is called its diameter.
- III. The largest vatar is called the diameter of the circle, and so on.

The following definitions in the Streometry course correspond to the definitions listed above:

- I₁. The geometric position (set of points) of points lying at the same distance from a given point A in space is called a sphere.
- II₁. The vein passing through the center of a sphere is called its diameter.
- III₁. The largest circumference of a sphere is called its diameter.

Thus, it can be seen from the above that based on the definitions of such concepts as "circle", "circle diameter", "circle diameter" given in the course "Planimetry" (similarly), in the course "Streometry" the concepts "sphere", "sphere diameter", "sphere diameter" definitions are given.

Similarly, [1,3,5]:

1. In accordance with the definition of "angle" in the course "Planimetry", the course "Stereometry" defines the concept of "angle between two half-planes".
2. Definition of the concept of "mutual parallelism of two straight lines" in the course "Planimetry" In the course "Stereometry" the definition of "parallelism of a straight line with two planes" is given.

3. According to the definition of the concept of "cross-sectional length" in the course "Planometry" in the course "Stereometry" are introduced the concepts of "surface", "volume".

If we approach the concept of analogy from a linguistic point of view, it corresponds to similar features of "qualitative concepts". Therefore, in the teaching of mathematics, it is necessary to teach students to distinguish "similar properties" in the mathematical objects under consideration. At the same time, it is important to ensure that students are able to perform tasks that help to determine which of the objects under consideration and what their properties correspond to the analogy, and to determine whether the resulting analogy is a mistake. Let's look at examples of assignments with such content below.

For example. 1). Based on the "sign of division of a number into 3 and 9" by analogy, it can be concluded that "the number is divisible by 27", ie "If the sum of the numbers of a number is divided by 27, then the number is divided by 27". But this conclusion is wrong. Because the number 272 745 is divisible by 3 and 9, but it is not divisible by 27. This means that the conclusion drawn by analogy is not always always true.

2) Teacher: If we increase the length of a straight rectangle by 2 times and decrease its length by 2 times, how much will its surface change?

Reader: It doesn't change. Indeed, the length of the neck of a given rectangle is a and the length of the width b if, $S=a \cdot b$ would be. In that case $S=2a \cdot \frac{b}{2} = a \cdot b$ will be.

Teacher: What if we increase the length of this right rectangle by 20% and decrease its width by 20%?

Pupil: It doesn't change.

Teacher: This conclusion would be a false analogy, because on the condition of the matter $x = a + 0,2 a \wedge y = b - 0,2 b \Rightarrow S = xy = (a + 0,2 a)(b - 0,2 b) = ab - 0,04ab = 0,96ab$ will be. Thus, the rectangular surface 4% decreased.

$$3) \frac{\lg 18}{\lg 9} = 2, \text{ because of this } \frac{18}{9} =$$

But this analogy is wrong. Here is the reader $\lg 18$, $\lg 9$ abbreviated, ignoring the fact that the connection is a symbol. However, in this case the result is as follows: $\frac{\lg 18}{\lg 9} = \frac{\lg(2 \cdot 9)}{\lg 9} = \frac{\lg 2 + \lg 9}{\lg 3} = \frac{\lg 2}{\lg 9} + \frac{\lg 9}{\lg 9} = \frac{\lg 2}{\lg 9} + 1 = 1 + \frac{\lg 2}{\lg 9}$.

4) Now let's look at the use of analogy in the task of "Determining the geometric position of points." It is known that in planimetry:

The geometric position of the points lying at the same distance from the ends of a given section consists of a perpendicular straight line drawn in the middle of that section.

The geometric position of points lying at the same distance from a given straight line consists of a straight line parallel to that straight line, the number of which is two.

In stereometry, the analogies to this are as follows:

The geometric position of the points lying at the same distance from the ends of a given section consists of a perpendicular plane passing through the middle of this section.

The geometric position of points at the same distance from a given plane consists of a plane parallel to that plane, the number of which is two.

in space (plane) AB the geometric position of the points where the difference of the squares of the distances from the end of the section is fixed is the perpendicular plane (straight line) passing through the middle of this section.

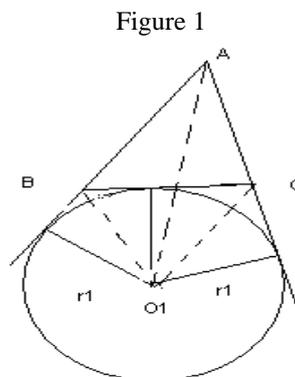
In general, in proving theorems using analogy, in the search for ways to solve problems, students are required to have the following knowledge, skills and abilities [4,9,10,]:

be able to choose a qualitative problem analogous to a given problem (or theorem);

be able to transfer the analysis to a given problem (or theorem) after solving (proving) the selected problem (or theorem), etc.

For example.

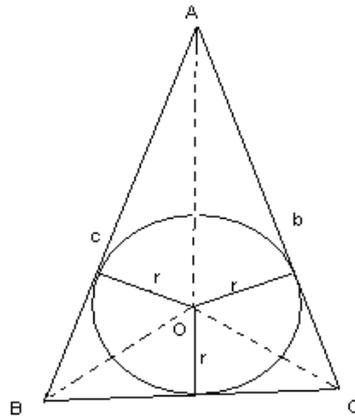
1) Find the radius of the circle (r_1) drawn inside a triangle with sides a , b and c , respectively (Fig.1) .



2) The radius of a circle drawn outside a triangle whose sides are a , b , and c , respectively (r_2) ни топинг (Fig.2).

Solution: Problem 1 can be solved using analogy only if problem 2 is solved first.

Figure 2

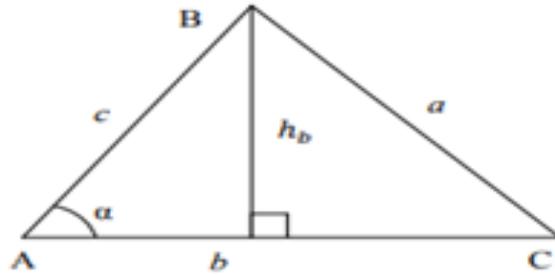


N ₂	Problem 1	Problem 2
1	$S_{\Delta ABC} = \sqrt{p(p-a)(p-b)(p-c)} \Rightarrow p = \frac{a+b+c}{2}$	$S_{\Delta ABC} = \sqrt{p(p-a)(p-b)(p-c)}$, in this $p = \frac{a+b+c}{2}$
2	$S_{\Delta AOB} = \frac{1}{2}cr_2 \wedge S_{\Delta AOC} = \frac{1}{2}br_2 \wedge S_{\Delta BOC} = \frac{1}{2}ar_2$	$S_{\Delta AO_1B} = \frac{1}{2}cr_1 \wedge S_{\Delta AO_1C} = \frac{1}{2}br_1 \wedge S_{\Delta BO_1C} = \frac{1}{2}ar_1$
3	$S_{\Delta ABC} = S_{\Delta AOB} + S_{\Delta AOC} + S_{\Delta BOC}$	$S_{\Delta ABC} = S_{\Delta AO_1B} + S_{\Delta AO_1C} - S_{\Delta BO_1C}$
4	$S = \frac{1}{2}(c+b+a)r = pr_2 \Rightarrow r = \frac{S}{p} \vee r_2 = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$	$S = \frac{1}{2}(c+b-a)r_1 = (p-a)r_1 \wedge (a=p-(c+b)) \Rightarrow r_1 = \frac{S}{p-a} \vee r_1 = \sqrt{\frac{p(p-b)(p-c)}{p-a}}$

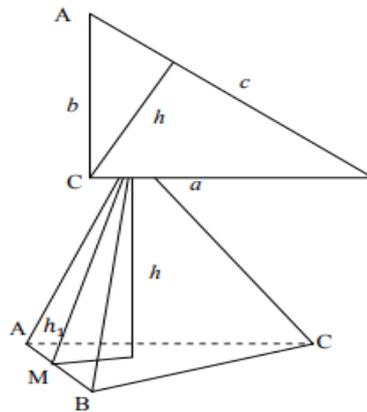
For example. The surfaces of the 2 sides of the tetrahedron are corresponding S_1 and S_2 equal. If the two-sided angle between them is α , and the length of the common side is a , $V = \frac{2S_1S_2 \sin \alpha}{3a}$ Prove that.

Proof. As is known, the triangle between the two sides and they face angle, which is equal to half the sum of sines $S_{ABC} = \frac{1}{2}cb \sin \alpha$ (1) will be. In that case, the graph ABH since it is a right-angled triangle, $h_b = c \sin \alpha$ (2) is the face of this triangle $S_{ABC} = \frac{1}{2}bh_b$ (3) will be. (2) if we put

(3), $S_{ABC} = \frac{1}{2} cbsina$ will be. Now, based on the above $V = \frac{2S_1S_2sina}{3a}$ we will develop a plan to prove the correctness of. It will be as follows:



- a) We represent the face of a given triangle ABC by one or more elements given in it. If we express through the b side, $S_{ABC} = \frac{1}{2} cbsina$ will be.
- b) We express the remaining elements using the given ones. For example. h_b we express the height using the c side and the chest. In that case, $h_b = csina$.
- c) If we put the results in parts a) and b) of the plan, $h_b = \frac{1}{2} cbsina$.
- d) If we put the results in parts a) and b) of the plan.



Based on the above, let us prove by the analogy method that $V = \frac{2S_1S_2sina}{3a}$ To do this, we first express the volume of the tetrahedron by the elements given in the problem condition. For example. Expressed over the surface, $V = \frac{1}{3} S_{ABC} h$.

If we define the face of an ADB side of a tetrahedron, will be

$$S_{ABD} = \frac{1}{2}ah_1 \text{ (5)}.$$

If we express the height h through the remaining known elements and the chest, will be $h = h_1 \sin \alpha$ (6). (6) and then (4) If we put, will be $V = \frac{1}{3}S_{ABC}h_1 \sin \alpha$ (7). If we express the height h_1 using the results (5) and (7): equal $h_1 = \frac{2S_{ABD}}{a} \Rightarrow V = \frac{1}{3}S_{ABC} \times \frac{2S_{ADB}}{a} \sin \alpha$.

Therefore, based on the above, it can be concluded that

$$V = \frac{2S_{ABC}S_{ADB}\sin\alpha}{3a}.$$

Now let's focus on the mistakes that can be made in the process of using analogy.

When pointing out mistakes in the use of analogy, it is advisable for the teacher to use the correct analogy in the given problem, to increase and deepen the knowledge of the error analogy, either directly or in collaboration with students, by solving inverse problems.

For example. The statement "-x is also a negative number because -7 is a negative number" is incorrect. To show this, x is the variable -7; - 8; 7; 8; 9; it is necessary to check the sign of -x by giving values. Without breaking the student's passion, initially supporting him $x = 7$, $x = 8$ in general

Convincing that $x = - 7$, $x = - 8$ is the opposite of the idea is appropriate, and that the final conclusion is reached by the students leads to a conscious division of concepts, i.e.

$$-x = \begin{cases} \text{positive number, } x < 0 \text{ if} \\ \text{negative number, } x > 0 \text{ if} \end{cases}$$

3. Conclusion

In general, the appropriate use of "Inference Methods" in the teaching of mathematics, in particular, the method of "Analogy" leads to the development of students' logical thinking skills, which in turn leads to conscious acquisition of knowledge and interest in learning mathematics.

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