

On a Covariant Functor ν in the Category R – of Compact Tychonov Spaces and their Continuous Mappings into Itself

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Abstract

This note defines a covariant functor $\mathcal{V}: Tych \rightarrow Tych$ acting on the category of Tychonov spaces and continuous mappings into itself. Studying the topological and categorical properties of this functor \mathcal{V} , it is shown that the functor \mathcal{V} is a normal functor in the category R – of compact spaces and continuous mappings into itself, which is a subcategory of $Tych$. It is proved that the functor $\mathcal{V}: Tych \rightarrow Tych$ is an open functor, in the considered category R – of compact spaces and continuous mappings into yourself.

For a Hewitt extension $\nu(X)$ of a Tychonov space X , the following holds:

a). $X \in Tych \Rightarrow \nu(X) \in Tych$;

- b). $\dim X = \dim \nu(X)$;
- c). $\nu(X)$ is the smallest R -compact subspace of $\beta(X)$ containing X ;
- d). $\nu(X)$ – is the largest subspace of βX in which X C – is embedding;
- e). $R - \omega(X) = \inf\{\tau : X \text{ } C\text{-embedding in } R^{\tau}\}$;
- e). $\omega(X) \subseteq R - \omega(X)$;
- f). $R - \omega(X) \subseteq |C(X)|$;
- g). $R - \omega(X) = R - \omega(\nu(X))$;
- h). $R - \omega(X) = \chi_0 \Leftrightarrow X$ is Polish space;
- i). If $X - C -$ is embedding in Y
 $Z - C -$ embedding in Y
 $X \subseteq Z \Rightarrow R - \omega(X) \subseteq R - \omega(Z)$
- j). If $f : X \rightarrow Y$ is a continuous mapping and Y is R -compact, then there is an extension $\tilde{f} : \nu(X) \rightarrow Y$ such that $\tilde{f}|_X = f$, that is;

$$\begin{array}{ccc} f : X & \rightarrow & Y - R - compact \\ & \searrow \tilde{f} & \\ & & \nu(X) \end{array}$$

- k). For arbitrary $\forall X \in Tych$

$$C(\nu(X))|_X = C(X)$$

If Y is a subspace of X , then $C(X)|_Y$ denotes the set of all elements of $C(Y)$ extending to the entire space X .

- l). If $X \in ANR$, then, by definition, for any C -, an embedding of X in an arbitrary Tychonoff space Y contains a functionally open neighborhood X in Y that retracts onto X , that is, leaving points of the space X fixed.
- m). If $X \in Tych$ and C -are embedded in Y , then $\nu(X)$ is also C -embedded in $\nu(Y)$.
- n). If $X \in Tych$ and R -are compact in Y , then $\nu(X)$ is closed in $\nu(Y)$.
- o). If $X \in AR$, then there is a Tychonoff space Y in which X C - is embedded and there is an $r : Y \rightarrow X$ retraction. Then $\nu(r) : \nu(Y) \rightarrow \nu(X)$ will also be a retraction.

Key-words: C - Embedding, R - Compactum, Functor, Continuous Extension.

1. Introduction

For a Tychonoff space X , we denote by $C(X)$ the space of continuous functions defined on X with a compact open topology.

It is known that a continuous mapping $f : X \rightarrow R$ is called a continuous (real) function. A set $U \subset X$ of a topological space X is called functionally open (respectively, functionally closed) if there exists a continuous function $f : X \rightarrow R$ such that $U = f^{-1}(0, +\infty)$ (respectively, $U = f^{-1}(0, +\infty)$) of a functionally open set is open. The complement of a functionally open set is functionally closed.

What has been said can be given a more precise meaning. Consider the category *Tych* of Tychonov spaces and their continuous mappings on the one hand, and the category *Vect* of linear spaces and their linear mappings on the other. The contravariant functor $C : Tych \rightarrow Vect$, which assigns to each space X a linear space $C(X)$ and to each continuous mapping $f : X \rightarrow Y$ an induced linear operator $C(f) : C(Y) \rightarrow C(X)$, allows us to distinguish a class of morphisms of the category *Tych* corresponding to epimorphisms of the category *Vect*. It is easy to see that $C(f)$ is epimorphic if and only if f is a C -embedding [1-3]. In other words, C -embeddings turn out to be “monomorphisms” in the category *Tych*.

Definition [1]. A Tychonov space X is called an absolute retract (respectively, an absolute neighborhood retract, notation: $X \in AR$) (respectively, notation: $X \in AR$) if for any C -embedding of X into an arbitrary Tychonov space Y (respectively, there is a functionally open neighborhood of X in Y) there is a retraction on X .

Recall that a functionally open neighborhood U of a set A in a space X is called stable if there is a set Z functionally closed in X such that $A \subseteq Z \subseteq U$. Note that if A is C -embedded in X , then any functionally open neighborhood U of A in X is stable [1]. It is well known that the retract of a Hausdorff space is closed in the space under consideration. That is why the space $X \in \mathcal{P}$ is called an absolute retractor in one or another class \mathcal{P} of Hausdorff spaces such that for any closed expression X into an arbitrary space $Y \in \mathcal{P}$ there is a retraction of Y onto X , that is, $r : Y \rightarrow X$ - retraction.

This definition of the concept of absolute retract has a significant logical flaw. The point is that the retracts are not only closed, but C – are embedded in the ambient space.

And by Uryson's theorem, C – embedding of closed subspaces characterizes normal spaces, and therefore the classical definition of the notion of the absolute is satisfactory only in relation to various subclasses of the class of normal spaces. However, already in the class of Tychonov spaces, this drawback is fully manifested. This can be confirmed by the result of Hanner [4-9] that every absolute retract in the class of Tikhonov spaces is compact. Therefore, with this definition, neither the Euclidean spaces R^n , nor the space R^ω are absolute retracts in the class of Tihanov spaces. (although they are such in the class of Polish spaces), which makes it impossible to use powerful methods of theory and retracts to study non-metrizable topological linear spaces and related questions of infinite-dimensional topology and functional analysis.

This raises the need for an additional requirement of C – embedding in the classical definition of the concept of an absolute retract. It turned out that in a number of cases the requirement of being closed can be discarded and limited to the requirement of C – embedding, since for spaces any C – embeddings are automatically closed.

Definition [1]. A space X is called an absolute (neighborhood) extensor in dimension n (abbreviated as $A(N)E(n)$ – space), $n = 0, 1, 2, \dots, \infty$, if for any space Z of dimension $\leq n$ and any subspace Z_0 , each map $f : Z_0 \rightarrow X$ such that $C(f) \subset C(X) \subseteq C(Z)|_Z$ can be extended to (some stable functionally open neighborhood Z_0 in Z) of the whole Z . $A(N)E(\infty)$ -space will be called absolute (neighborhood) extensors, or in short $A(N)E$ - spaces [10-12].

Proposition [1]. a) The class of $A(N)R$ -spaces coincides with the class of $A(N)E$ -spaces.

b) Every $ANE(0)$ -space is an $AE(0)$ -space.

c) every $ANE(n)$ -space is an $AE(0)$ -space.

d) Every $AE(0)$ -space R -compact.

2. Main Part

For a Tychonov space X , we consider a really compact extension $V(X)$. This $V(X)$ extension of X is given in two ways. On the space $C(X)$ with the compact-open topology, we take

a diagonal mapping of its elements, which is an embedding of the space X onto the space $R^{C(X)}$, that is,

$$f = \Delta\{f_\alpha(x) : f_\alpha(x) \in C(X)\}$$

Studying the topological properties of the space $V(X)$, we prove that the mapping $V : X \rightarrow V(X)$ is functorial. Further, we show that $V : Tych \rightarrow Tych$ is a covariant functor satisfying a number of properties of a normal functor in the sense of Shepin. Most importantly, the $V(X)$ R -space is compact and closed in $R^{C(X)}$.

Consider the canonical embedding $i_X = \Delta\{\varphi : \varphi \in C(X)\} : X \rightarrow R^{C(X)}$ which is the diagonal product of all mappings of the space $C(X)$ and identify the space X with its image $i_X(X)$. We denote the closure of X in $R^{C(X)}$ by $V(X)$ that is, $V(X) = \overline{i_X(X)} = \Delta\{\varphi : \varphi \in C(X)\}(X) \subset R^{C(X)}$, $V(X) = \overline{\Delta(X)}$. Obviously, $\overline{V(X)} \subset R^{|C(X)|} = R^\tau$, where $|C(X)| = \tau$. This means that the space $V(X)$ is closed in R^τ . The product of real numbers R^τ , which is Tychonov's space. The space $V(X)$ is called a Hewitt extension, since the space X is everywhere dense in $V(X)$.

This means that $V(X)$ is also a Tychonov space, since $V(X)$ is a closed subspace of a Tychonov space R^τ .

Definition [1]. A space X is called R -compact if it is homeomorphic to a closed subspace of some power of the real line.

The definition of a space implies that $V(X)$ is an R -compact space.

A subspace X is said to be C -embedded in a space Y if $C(X)|_Y = C(Y)$. and by $C(X)|_Y$ - denote the set of all mappings $f : Y \rightarrow R$ extendable to all X elements of $C(Y)$.

From Proposition 3.2.5 [1] it immediately follows that R -compact spaces are completely characterized by the equality $X = V(X)$.

Definition [1]. A subspace X of a space Y is called Z -embedded if every functionally open subset G of X is representable as an intersection $G = X \cap V$, where V is a functionally open subset of Y .

Another, often more convenient construction of the Hewitt extension $\mathcal{V}(X)$ can be obtained as follows. Assuming the space $X \subseteq Z$ to be an embedded subspace of some R -compactum Y (for example, a stone of the Stoyan-Chekhov extension βX), it suffices to consider the intersection of all functionally open sets Y containing X . From the proof of 3.2.13 [1] it is clear that X is everywhere dense and C -embedded in this intersection that is R -compact by Corollary 3.2.3 [1]. Thus, the indicated intersection can serve as one of the models of the extension $\mathcal{V}(X)$. Hence, we can assume that the inclusions $X \subseteq \mathcal{V}(X) \subseteq \beta X$ are fulfilled. Hence, from this we can also conclude that:

a) $\mathcal{V}(X)$ -smallest R -compact subspace $\beta(X)$ containing X ;

b) $\mathcal{V}(X)$ is the largest subspace βX in which C is embedded;

Equality immediately follows from the second construction.

c) $\dim X = \dim \mathcal{V}(X)$;

$\dim X$ denotes the dimension of the space X in the sense of porosity.

Hence, the space $\mathcal{V}(X)$ is C -embedded and R -compact in $R^{C(X)}$.

Let $f : X \rightarrow Y$ be a continuous surjective mapping between Tychonoff spaces, that is, $f(X) = Y$, $X \in Tych$ and $Y \in Tych$.

Consider a mapping $\mathcal{V}(f) : \mathcal{V}(X) \rightarrow \mathcal{V}(Y)$ that satisfies the condition $\mathcal{V}(f) = \beta(f)|_{\mathcal{V}(X)}$ where $\beta(f) : \beta(X) \rightarrow \beta(Y)$. Let us associate each $X \in Tych$ with its Hute extension $\mathcal{V}(X)$, i.e. $X \rightarrow \mathcal{V}(X)$ where $\mathcal{V}(X)$ is R -compact and C -embedded in $R^{C(X)}$.

On the other hand, each mapping $f : X \rightarrow Y$ can be associated with a well-defined mapping $\mathcal{V}(f) : \mathcal{V}(X) \rightarrow \mathcal{V}(Y)$, which coincides with the mapping f to X .

The definition of this mapping follows from the proof of the following fact.

Assumption 3.2.5 [1]. R -compact and C -embedded subspaces are closed.

Proof. Let X – R -compact and C -embedded subspace Y . By the definition of R -compact, there exist a set A and a closed subspace X' in R^A such that there exists a homeomorphism f of the space X onto X' , denote by $\pi_\alpha : R^A \rightarrow R_\alpha$ the projection onto the corresponding factor.

Since X is C -embedded in Y , for each $\alpha \in A$ there is a function $\varphi_\alpha \in C(Y)$ such that $\varphi_\alpha|_X = \pi_\alpha \circ f$. Put $\varphi = \Delta\{\varphi_\alpha : \alpha \in A\}$. It is clear that φ maps the space Y to R^A and $\varphi|_X = f$.

In this case, we get the following diagram:

$$\begin{array}{ccc}
 X' \subset R^A = \prod_{\alpha \in A} R_\alpha, & R_\alpha = R & \\
 \begin{array}{c} \nearrow \varphi \\ \leftarrow f \\ \leftarrow \varphi_\alpha \end{array} & & \begin{array}{c} \searrow \pi_\alpha \\ \downarrow \pi_\alpha \\ \leftarrow \varphi_\alpha \end{array} \\
 X \xrightarrow{\varphi} R & & \\
 \begin{array}{c} \subset \\ \leftarrow \varphi_\alpha \end{array} & & Y \xrightarrow{\varphi_\alpha} R
 \end{array}
 \tag{1}$$

$$\varphi_\alpha|_X = \varphi_\alpha$$

From the continuity of φ and the closedness of X' in R^A , we obtain

$$\varphi([\varphi]_Y) \subseteq [\varphi(X)]_{R^A} = [f(X)]_{R^A} = [X']_{R^A} = X' \tag{2}$$

In other words, the $g = \varphi|_{[X]_Y} = \varphi|_{[X]_Y}$ constraint maps the closure of $[X]_Y$ to X' , i.e.

$$G:[X]_Y \rightarrow X'.$$

Since f is a homeomorphism, then there is definitely a mapping $r = f^{-1} \cdot g : [X]_Y \rightarrow X$, which is easy to verify that it is a retraction. Consequently, $X = [X]_Y$. The proposition is proved.

A linear operator $U : C(X) \rightarrow C(Y)$ is called regular if the following conditions are satisfied:

- (i) the mapping $U : C(X) \rightarrow C(Y)$ is continuous;
- (ii) if $\varphi \geq 0$, then $U(\varphi) \geq 0$ (that is, the operator is positive);
- (iii) $U(1_X) = 1_Y$, where $1_X : X \rightarrow \{1\} \subset \mathbf{R}$ is a constant function, i.e. the operator translates constant functions into constant ones.

Each continuous mapping $f : X \rightarrow Y$ generates a regular operator $f^* : C(Y) \rightarrow C(X)$ by the formula $f^*(\varphi) = \varphi \circ f$, where $\varphi \in C(Y)$.

If X is closed in Y and for every $\varphi \in C(X)$ the restriction of the function $U(\varphi)$ to X coincides with φ , then the operator U is called the extension operator.

If the mapping $f : X \rightarrow Y$ is surjective, then the regular operator $U : C(X) \rightarrow C(Y)$ is called the regular averaging operator [2].

If Y is a subspace of X , then the symbol $C(X)|_Y$ denotes the set of all elements from $C(Y)$ extendable to all X .

On the other hand, for the $f : X \rightarrow Y$ display, the following diagram takes place:

$$\begin{array}{ccc}
X & \xrightarrow{f} & Y \\
\downarrow \Delta \phi_\alpha^X & & \downarrow \phi_\beta^Y \\
R_\alpha & \xrightarrow{\Delta f} & R_\alpha = R \\
\uparrow \pi_\alpha & & \uparrow \pi_\alpha \\
R^{C(X)} & \xrightarrow{\Delta f} & R^{C(X)}
\end{array}
\quad (3)$$

It follows from the pre-zone fact and from this diagram that the mapping $V(f)$ is completely definite and satisfies the condition

$$V(f)|_X = f. \quad (4)$$

Note that $V: Tych \rightarrow Tych$ since if $X \in Tych$ then $V(X) \in Tych$ and $f \in Tych$ then $V(f): V(X) \rightarrow V(Y)$ continuous mapping

a) if $f, g \in M$ then $f \circ g \in M$, i.e.

$$\begin{array}{ccc}
X \xrightarrow{f} Y & & V(X) \xrightarrow{V(f)} V(Y) \\
g \circ f \searrow \swarrow g & \xrightarrow{V} & V(g) \searrow \swarrow V(g) \\
Z & & V(Z)
\end{array}
\quad (5)$$

This means that the functor V maps commutative diagrams to commutative ones.

b) $V(id_X) = id_{V(X)}$ for any $X \in Tych$.

Hence $V: Tych \rightarrow Tych$ is a covariant functor in the category $Tych$ and continuous mappings into itself.

Let $f: X \rightarrow Y$ be a monomorphism. By definition, $V(f): V(X) \rightarrow V(Y)$ is also a monomorphism.

If $f: X \rightarrow Y$ is surjective, then $V(f): V(X) \rightarrow V(Y)$ is surjective.

Note that if X is compact then $V(X)$ is compact and $V(X) \cong X$.

On the other hand, $V(\{\emptyset\}) = \emptyset$ and $V\{1\} = 1$ point.

The spectrum $S = \{X_\alpha, \pi_\beta^\alpha, A\}$ is said to be continuous if, for every limiting element γ , there exists a limiting mapping $\lim\{\pi_\alpha^\gamma: \alpha < \gamma\}: X \rightarrow \lim(S|_\gamma)$ is a homeomorphism.

Let $S = \{X_\alpha, \pi_\beta^\alpha, A\}$ be the inverse spectrum. We put

$\mathcal{V}(S) = \{\mathcal{V}(X_\alpha); \mathcal{V}(\pi_\beta^\alpha), A\}$. Then $\mathcal{V}(S)$ is also the inverse spectrum. Let us denote its through projections $\pi_\alpha^\mathcal{V}$.

A functor \mathcal{V} is called continuous if $F(\lim S) = \lim F(S)$ for any spectrum S . Moreover, this means that there exists a homeomorphism $f : F(\lim S) = \lim F(S)$ for which

$$F(\pi_\alpha) = \pi_\alpha^F \circ f \quad (6)$$

Note that a mapping f satisfying condition (1) always exists and is unique. This is the limit of $F(\pi_\alpha)$ mappings. In other words, the functor F is continuous if the limit of the mappings is $F(\pi_\alpha)$ -homeomorphism.

It follows immediately from the definition of the continuous spectrum and the above properties of the functor \mathcal{V} .

Proposition 1. The functor $\mathcal{V} : Tych \rightarrow Tych$ is continuous.

The functor F is called open; it takes open mappings into open ones [3].

Proposition 2. The functor \mathcal{V} is an open functor.

Proof. Let $f : X \rightarrow Y$ be an open mapping. Consider the mapping $\mathcal{V}(f) : \mathcal{V}(X) \rightarrow \mathcal{V}(Y)$.

Let $v_0^x \in \mathcal{V}(X)$ and $\mathcal{V}(f)(v_0^x) = v_0^y \in \mathcal{V}(Y)$, where $v_0^x \in \mathcal{V}(X) \subset R^A$ and $v_0^y \in \mathcal{V}(Y) \subset R^B$,

$$\mathcal{V}(X) = \overline{\Delta(X)} \subset R^A \quad \mathcal{V}(X) \text{ closed in } R^A$$

$$\mathcal{V}(Y) = \overline{\Delta(Y)} \subset R^B, \quad \mathcal{V}(Y) \text{ closed in } R^B.$$

It follows from the diagram (3) - (6) and the definition of the spaces $\mathcal{V}(X)$ and $\mathcal{V}(Y)$ that the openness of the mapping $\mathcal{V}(f)$ is easy to check. Proposition 2 is proved.

A functor is called pre-image preserving if

$$F(f)^{-1}(F_Y(A)) = F_X(f^{-1}A) \quad (7)$$

for any mapping $f : X \rightarrow Y$ and any closed $A \subset Y$.

Let $f : X \rightarrow Y$ be a continuous mapping between Tychonov spaces X and Y .

We noticed that if $A \subset X$ then $\mathcal{V}(A) \subset \mathcal{V}(X)$ where, A is closed in X .

Let A be closed in Y and AY , then VA . A direct check shows that the equality

$$\mathcal{V}(f)^{-1}(\mathcal{V}_Y(A)) = \mathcal{V}_X(f^{-1}(A)) \quad (8)$$

This means that the functor preserves the inverse images of the mappings.

An R -weight of a space X is a minimal infinite cardinal τ such that there exists a C-embedding of X in R^τ - (notation $R - \omega(X) = \tau$).

In this case, a C-embedding is understood as an embedding with a C-embedded image.

Obviously, the weight of an arbitrary space does not exceed its R -weight. For infinite compacta, the converse is also true.

It is easy to verify that the following two facts are true:

- a) $R - \omega(X) \leq |C(X)|$; (9)
- b) $R - \omega(X) = R - \omega(\nu(X))$;

The countability of the R -weight of some space X is equivalent to the fact that X is a complete metric space with a countable base. Obviously, the monotonicity of the R -weight with respect to the C-embedded subspace. From here, we can write

$$\omega(X) = \omega(\nu(X)). \quad (10)$$

Thus, we have shown that the functor $\nu: Tych \rightarrow Tych$ in the category of Tychonov spaces and their continuous mappings into itself is a covariant normal functor in the sense of Shepin [10-12].

Theorem. Let X be a Tychonov space.

$$\begin{aligned} X \in A(N)E &\Leftrightarrow \nu(X) \in A(N)E \\ X \in A(N)E(n) &\Leftrightarrow \nu(X) \in A(N)E(n) \\ X \in A(N)R &\Rightarrow \nu(X) \in A(N)R; \\ X \in ANE(0) &\Rightarrow \nu(X) \in ANE(0); \\ X \in ANE(n) &\Rightarrow \nu(X) \in AE(0); \\ X \in AE(0) &\Rightarrow X - R - \text{-compact}; \end{aligned}$$

$X \subset Y$ there is $r: Y \rightarrow X$ - retraction, arbitrary $Y \in Tych$ $\nu(r): \nu(Y) \rightarrow \nu(X)$ - retraction of arbitrary $\nu(X), \nu(X)$.

Proof. Let $X \in Tych$, $X \in A(N)R$ and X be infinite. From the definition, the space $\nu(X)$ is R -compact. Let us prove the neighborhood version. The absolute version is proved in a similar way. Consider an arbitrary space Z of dimension $\dim Z \leq n$, an arbitrary subspace Z_0 , and

a mapping $f : Z_0 \rightarrow \nu(X)$ with the $C(f) (C(X)) \subset C(Z)|_{Z_0}$ property. By A_0 we denote the closure of the set Z_0 in the Hewitt extension $\nu(Z)$ of the space Z , that is, $A_0 = \nu(Z_0) \subset \nu(Z)$. Since $Z - C -$ is embedded in $\nu(Z)$, the inclusion of $C(f) \subset C(X) \subseteq C(\nu Z)|_{Z_0}$. By virtue of Proposition 3.2.24 [1], the mapping f can be extended to the mapping $g : A_0 \rightarrow \nu(X)$ satisfying the relation $C(g)C(X) \subseteq C(\nu(Z))|_{A_0}$. Since $\dim \nu(Z) = \dim Z \leq n$ it follows from Proposition 3.3.9 [1] that there exists a stable functionally open neighborhood V in $\nu(Z)$ of the set A_0 and a map $h : V \rightarrow X$ such that $h|_{A_0} = g$. It remains to note that the set $U = V \cap Z$ is a stable functionally open neighborhood of the set Z_0 in Z and the mapping $h|_U -$ is an extension of the original mapping. The theorem is proved.

Note the following properties of the Hewitt extension $\nu(X)$ of a Tychonoff space X :

- a). $X \in Tych \Rightarrow \nu(X) \in Tych$;
- b). $\dim X = \dim \nu(X)$;
- c). $\nu(X)$ is the smallest $R -$ compact subspace of $\beta(X)$ containing X ;
- d). $\nu(X) -$ is the largest subspace of βX in which $X - C -$ is embedded;
- e). $R - \omega(X) = \inf\{\tau : X - C - \text{embedding in } R^\tau\}$;
- e). $\omega(X) \subseteq R - \omega(X)$;
- f). $R - \omega(X) \subseteq |C(X)|$;
- g). $R - \omega(X) = R - \omega(\nu(X))$;
- h). $R - \omega(X) = \chi_0 \Leftrightarrow X$ is Polish space;
- i). If $X - C -$ is embedding in Y
 $Z - C -$ embedding in Y
 $X \subseteq Z \Rightarrow R - \omega(X) \subseteq R - \omega(Z)$
- j). If $f : X \rightarrow Y$ is a continuous mapping and Y is $R -$ compact, then there is an extension $\tilde{f} : \nu(X) \rightarrow Y$ such that $\tilde{f}|_X = f$, that is;

$$\begin{array}{ccc}
 f: X & \rightarrow & Y - R - \text{compact} \\
 & \tilde{f} & \\
 & \nu(X) &
 \end{array}$$

k). For arbitrary $\forall X \in Tych$

$$C(\nu(X))|_X = C(X)$$

If Y is a subspace of X , then $C(X)|_Y$ denotes the set of all elements of $C(Y)$ extending to the entire space X .

l). If $X \in ANR$, then, by definition, for any $C-$, an embedding of X in an arbitrary Tychonoff space Y contains a functionally open neighborhood X in Y that retracts onto Y , that is, leaving points of the space X fixed;

m). If $X \in Tych$ and $C-$ are embedded in Y , then $\nu(X)$ is also $C-$ embedded in $\nu(Y)$;

n). If $X \in Tych$ and $R-$ are compact in Y , then $\nu(X)$ is closed in $\nu(Y)$;

o). If $X \in AR$, then there is a Tychonoff space Y in which X $C-$ is embedded and there is an $r: Y \rightarrow X$ retraction. Then $\nu(r): \nu(Y) \rightarrow \nu(X)$ will also be a retraction.

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