Non Homogeneous Rough Finite State Automaton

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Abstract

Objective: *The study of finite state automaton is an essential tool in machine learning and artificial intelligence. The class of rough finite state automaton captures the uncertainty using the rough transition map. The need to generalize this concept arises to adhere the dynamical behaviour of the system. Hence this paper focuses on defining non-homogeneous rough finite state automaton.* ***Methodology:*** *With the aid of Rough finite state automata we define the concept of non-homogeneous rough finite state automata.* ***Findings:*** *Non homogeneous Rough Finite State Automata (NRFSA) M_t is defined by a tuple $(Q, \Sigma, \delta_t, q_0(t), F(t))$ The dynamical behaviour of any system can be expressed in terms of an information system at time t. This leads us to define non-homogeneous rough finite state automaton. For each time 't' we generate lower approximation rough finite state automaton M^t_- and the upper approximation rough finite state automaton M^{t^-} and the defined concepts are elaborated with suitable examples. The ordered pair $M_t = (M(t)_-, M(t)^-)$ is called as the non-homogeneous rough finite state automaton.* ***Conclusion:*** *Over all our study reveals the characterization of the system which changes its behaviour dynamically over a time 't'.* ***Novelty:*** *The novelty of the proposed article is that it clearly immense the system behaviour over a time 't'. Using this concept the possible and the definite transitions in the system can be calculated in any given time 't'.*

1. Introduction

Rough set theory is an efficient tool to deal with an uncertainty and Automata theory is an another important tool in machine learning. In our previous research article[1] we clubbed this two

efficient tool an defined a new class of automaton called rough finite state automaton. But it is not sufficient to deal with the dynamical behaviour of the system this leads us to define a new concept called Non-homogeneous Rough Finite State Automaton (NRFSA).

In [4] the authors applied the concept of CFS to homogeneous finite state automaton and deals in reduction of the transition in automaton. In [9] The authors discussed the properties of RFSA and the product of RFSA is introduced. In [3] the author investigate the transition reduction in DFA. Many researchers worked with rough finite state automaton [2] [5] [6] [7] [8][9][10] and investigated their properties using different types of tool.

In section 2 some of the basic definitions are discussed to study the rest of the paper. In section 3 non-homogeneous information system with decision variables are discussed and we illustrate the concepts with suitable examples. In section 4 we define Non-homogeneous Rough Finite State Automaton and the language for each time t is discussed with suitable examples.

2. Preliminaries

In this section some of the basic definitions of Automata theory and Rough set theory is described.

Definition 2.1. (Finite Automaton)

An Automaton is represented by 5-tuples $(Q, \Sigma, \delta, q_0, F)$, where Q is a nonempty finite set of states, Σ is a finite set of symbols called the alphabet of the automata, δ is the transition function, q_0 is the initial state from where any input is processed ($q_0 \in Q$) and F is the set of final state/states of Q ($F \subseteq Q$).

Definition 2.2. (Rough Set)

The data analysis based on the concept Rough set will start from the information table. The information table $I = (U, A)$ will contain rows (objects) and column (attributes) which is defined by $\mu_a : U \rightarrow [0, 1]$, $a \in A$ is a fuzzy set. Any set $P \in A$ determines a binary relation I(P) on U, which is called as an indiscernibility relation, and defined as follows: $I(P) = \{(x, y) \in U^2 \mid \forall a \in P, \mu_a(x) = \mu_a(y)\}$. The partition induced by I(P) consists of equivalence classes defined by $[x]_p = \{y \in U | (x, y) \in I(P)\}$. For any for $X \subseteq U$, define the lower approximation space

P_- (X) = {x ∈ U|[x]$_p$ ⊆ X} and the upper approximation P^-(X) = {x∈U|[x]$_p$ ∩ X≠ϕ}. Then the rough set corresponding to X in the approximation space P, we mean the ordered pair RS(X) = (P_- (X), P^-(X)).

Definition 2.3. (Rough Finite State Automata)

The Rough Finite State Automata (RFSA) is a five tuple M = (Q, Σ, δ, q$_0$, F) Where, Q is a nonempty finite set of states, Σ is the finite set Input Symbols, q$_0$ is the initial state q$_0$ ∈ Q, F ⊆ Q is the set of final states. Then the rough transition map δ is defined by, δ : Q × Σ → ℙ(Q) × ℙ (Q). In Rough Finite State Automata δ can be described as a pair of functions ($δ_-$, $δ^-$).

The RFSA M can be viewed as a pair of functions M = (M_-, M^-). Where M_- is called as the Lower Approximation Automata and M^- is called as the Upper Approximation Automata induced by the rough transition map δ.

Definition 2.4. (Language generated by RFSA)

The language generated by the Rough Finite State Automata M is denoted by L(M), where
L(M) = ($L(M)_-$, $L(M)^-$)

$L(M)_-$ is the language generated by the lower approximation automata M_- and $L(M)^-$ is the language generated by the upper approximation automata M^-.

3. Non Homogeneous Information System Induced by Decision Variables

Consider an information system $I_t = (U, A)$ be an information system, where U is a nonempty finite set of objects and A is the nonempty finite set of fuzzy attributes. R is an indiscernibility equivalence relation on U. Let N be the finite set of decision variables $N = \{a, b, \ldots, s\}$ and $\forall x ∈ U, \mu_a(t) : U → [0, 1]$ such that $\mu_a(t)(x_i)$ gives the grade of membership for the object x_i posses the decisions a, and $\forall a ∈ N, \mu_a(t)(x) : U → [0, 1]$ such that $\mu_a(t)(x_i)$ gives the grade of membership for the object x_i for accepting the decision a and also $\forall a ∈ N, \eta_a(t) : U → ℙ(U)$ be the neighbourhood function such that for any $x ∈ U,, \eta_a(t)(x)$ contains those elements of U, having almost the same opinion with respect to the decision a.

Hence $\forall a ∈ N, x ∈ U, \eta_a(t)(x) ⊆ U$ and hence we can have the corresponding Rough set
$$RS(\eta_a(t)(x)) = (\eta_a(t)(x)_-, \eta_a(t)(x)^-)$$
Where,

$$\eta_a(t)(x)_{-} = \{y \in U \,|\, [y]_p \subseteq \eta_a(t)(x)\} \text{ and } \eta_a(t)(x)^{-} = \{y \in U \,|\, [y]_p \cap \eta_a(t)(x) \neq \phi\}.$$

Here $\eta_a(t)(x)_{-}$ contains all equivalence classes induced by R that are completely contained in $\eta_a(t)(x)$. In fact, if a class is in $\eta_a(t)(x)_{-}$ then those element are having similar attribute values and they are in the neighbourhood of x with respect to decision, similarly if a class is in $\eta_a(t)(x)_{-}$ then those elements are possibly be the member of the neighbourhood of x with respect to decision a.

This Rough set is called as Rough set induced by Decision variable a.

$$R_*(t)(x) = \bigcap_{a \in N} \eta_a(t)(x)_{-} \quad R_*(t)(a) = \bigcap_{x \in U} \eta_a^t(x)_{-}$$

$$R^*(t)(x) = \bigcap_{a \in N} \eta_a^t(x)^{-} \quad R^*(t)(a) = \bigcap_{x \in U} \eta_a^t(x)^{-}$$

Here, $R_*(t)(x)$ is the set of elements of U that are in the neighbourhood of x, satisfy all the decision variables at time t and $R^*(t)(x)$ is the set of elements of U that are possibly in the neighbourhood of x, satisfy all the decision variables at time t. $R_*(t)(a)$ contains all elements of U, satisfying the decision variables a and $R^*(t)(a)$ contains all possible elements of U satisfying the decision variable a at time t. Throughout this paper, we consider an Information System $I_t = (U, A, N)$ where U is a nonempty finite set of objects, A is the nonempty finite set of fuzzy attributes, N is the set of decision variables and we call this information system as a Information System with decision variables.

Example 3.1. The information system with Decision variables a and b is given by the following table. (table 1)

$U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and N = {a,b}. The equivalence classes are

$X_1 = \{x_1, x_3\}$

$X_2 = \{x_2, x_4, x_6\}$

$X_3 = \{x_5\}$

Table 1 - Information System with Decision Variables

A/D/U	a_1	a_2	a_3	a_4	a	b
x_1	0	0.1	0.3	0.2	0.4	0.5
x_2	1	0.6	0.7	0.3	0.2	0.7
x_3	0	0.1	0.3	0.2	0.8	0.4
x_4	1	0.6	0.7	0.3	0.7	0.6
x_5	0.8	0.5	0.2	0.4	0.6	0.2
x_6	1	0.6	0.7	0.3	1	1

Let us take $\delta_a = \delta_b = 0.2, \forall a \in N, \eta_a(t): U \to \mathbb{P}(U)$ be the neighbourhood function, such that,

$$\eta_a(1)(x) = \{y \in U || \mu_a(1)(x) - \mu_a(1)(y)| \le 0.2\},$$
$$\eta_a(1)(x_1) = \{x_1, x_2, x_5\}$$
$$\eta_a(1)(x_2) = \{x_1, x_2\}$$
$$RS(\eta_a(1)(x_1)) = \{X_3, U\}, RS(\eta_a(1)(x_2)) = \{\phi, X_1 \cup X_2\}$$

and for the decision variable b,

$$\eta_b(1)(x) = \{y \in U || \mu_b(1)(x) - \mu_b(1)(y)| \le 0.2\}$$
$$\eta_b(1)(x_1) = \{x_1, x_2, x_3, x_4\}$$
$$\eta_b(1)(x_2) = \{x_1, x_2, x_4, x_6\}$$
$$RS(\eta_b(1)(x_1) = \{X_1, X_1 \cup X_2\}, RS(\eta_b(1)(x_2) = \{X_2, X_1 \cup X_2\}$$

Similarly for all $x \in U$, the corresponding Rough set are calculated and depicted in the following table (Table 2).

Table 2 - Lower and Upper Approximation of the Decision Variables

η_a/U	$\eta_a(1)_-$	$\eta_a(1)^-$	$\eta_b(1)_-$	$\eta_b(1)^-$
x_1	X_3	U	X_1	$X_1 \cup X_2$
x_2	ϕ	X_3	X_2	$X_1 \cup X_2$
x_3	X_3	U	$X_1 \cup X_3$	U
x_4	X_3	U	X_1	$X_1 \cup X_2$
x_5	X_3	U	X_3	$X_1 \cup X_3$
x_6	ϕ	X_3	ϕ	X_2

By definition, the values of $R_*(1)(a), R^*(1)(a), R_*(1)(x_j)$ and $R^*(1)(x_j)$ are depicted in the following table 3 and table 4 respectively. $R_*(1)(a) = \phi$ implies none of the elements of U are satisfying the decision variable a and $R^*(1)(a) = X_1 \cup X_2$ implies that the elements of $X_1 \cup X_2$ are possibly satisfies the decision variable a. But for the decision variable b, $R_*(1)(b) = R^*(1)(b) = \phi$ implies that definitely all the member of U are not satisfying the decision variable b.

Table 3 - $R_*(1)$ and $R^*(1)$ for the Decision Variables of the Information System

R\D	$R_*(1)$	$R^*(1)$
a	ϕ	$X_1 \cup X_2$
b	ϕ	ϕ

$R_*(1)(x_3) = X_3$ implies that the element of X_3 are in the neighbourhood of x_3 with respect to both of the decision variables a and b.

$R^*(1)(x_3) = U$ implies that all the elements of U are in the neighbourhood of x_3 with respect to both of the decision variables a and b.

Table 4 - $R_*(1)$ and $R^*(1)$ for all the Elements of U

U\R	x_1	x_2	x_3	x_4	x_5	x_6
$R_*(1)$	ϕ	ϕ	X_3	ϕ	X_3	ϕ
$R^*(1)$	$X_1 \cup X_2$	$X_1 \cup X_2$	U	$X_1 \cup X_2$	$X_1 \cup X_3$	X_2

Consider an information system at time t = 2, (table.5)

Table 5 - Information System with decision variables

A/D/U	a_1	a_2	a_3	a_4	a	b
x_1	0	0.1	0.3	0.2	0.4	0.5
x_2	0.9	0.8	0.6	0.4	0.2	0.7
x_3	0	0.1	0.3	0.2	0.8	0.4
x_4	1	0.6	0.7	0.3	0.7	0.6
x_5	0.8	0.5	0.2	0.4	0.6	0.2
x_6	1	0.6	0.7	0.3	1	1

$U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and N = {a,b}. The equivalence classes are

$X_1 = \{x_1, x_3\}$

$X_2 = \{x_2\}$

$X_3 = \{, x_4, x_6\}$

$X_3 = \{x_5\}$

Let us take $\delta_a = \delta_b = 0.2$;

$\eta_a(2)(x) = \{y \in U | |\mu_a(2)(x) - \mu_a(2)(y)| \leq 0.2\}$,

By definition, the values of $R_*(2)(a)$, $R^*(2)(a)$, $R_*(2)(x_j)$ and $R^*(2)(x_j)$ are depicted in the following table 6 and table 7 respectively.

Table 6 - $R_*(2)$ and $R^*(2)$ for the Decision Variables of the Information System

R\D	$R_*(2)$	$R^*(2)$
a	ϕ	$X_1 \cup X_2$
b	ϕ	ϕ

Table 7 - $R_*(2)$ *and* $R^*(2)$ for all the Elements of U

U\R	x_1	x_2	x_3	x_4	x_5	x_6
$R_*(2)$	ϕ	ϕ	X_3	ϕ	X_3	ϕ
$R^*(2)$	$X_1 \cup X_2$	$X_1 \cup X_2$	U	$X_1 \cup X_2$	$X_1 \cup X_3$	X_2

Table 8 - Lower and Upper Approximation of the Decision Variables

η_a/U	$\eta_a(2)_-$	$\eta_a(2)^-$	$\eta_b(2)_-$	$\eta_b(2)^-$
x_1	X_3	$X_1 \cup X_2 \cup X_3$	$X_1 \cup X_2$	$X_1 \cup X_2 \cup X_3$
x_2	X_2	$X_1 \cup X_2$	$X_2 \cup X_3$	$X_1 \cup X_2 \cup X_3$
x_3	X_4	$X_1 \cup X_3 \cup X_4$	$X_1 \cup X_4$	$X_1 \cup X_3 \cup X_4$
x_4	X_4	$X_1 \cup X_3 \cup X_4$	$X_1 \cup X_2$	$X_1 \cup X_2 \cup X_3$
x_5	$X_1 \cup X_4$	$X_1 \cup X_3 \cup X_4$	X_4	$X_1 \cup X_4$
x_6	ϕ	$X_1 \cup X_3$	ϕ	X_3

4. Non-homogeneous Rough Finite State Automata Induced by Information System with Decision Variables

In this section, we consider an Information System with the decision variables $I_t = (U, A, N)$.

Definition 4.1. (Non-Homogeneous Rough Finite State Automata)

Non-homogeneous Rough Finite State Automata (NRFSA) is defined by a five tuple $M_t = (Q, \Sigma, \delta_t, q_0(t), F(t))$ Where Q is a nonempty finite set of states, Σ is the finite set Input Symbols, $q_0(t)$ is the initial state $q_0 \in Q$, $F(t) \subseteq Q$ is the set of final states. Then the rough transition map δ_t is defined by, $\delta_t : Q \times \Sigma \rightarrow \mathbb{P}(Q) \times \mathbb{P}(Q)$.

In Non-homogenous Rough Finite State Automata δ_t can be described as a pair of functions $(\delta(t)_-, \delta(t)^-)$.

The NRFSA M_t can be viewed as a pair of functions $M_t = (M(t)_-, M(t)^-)$. where $M(t)_-$ is called as the Lower Approximation Automata and $M(t)^-$ is called as the Upper Approximation Automata induced by the rough transition map δ_t..

Definition 4.2. (Language Generated by N`RFSA)

Refer definition 2.2 in [1] The language generated by the Non-homogeneous Rough Finite State Automata M_t is denoted by $L(M_t)$, where $L(M_t) = (L(M)(t)_-, L(M)(t)^-)$

$L(M)(t)_-$ is the language generated by the lower approximation automata $M(t)_-$ and $L(\text{M})(\text{t})^-$ is the language generated by the upper approximation automata $\text{M}(\text{t})^-$.

$$L(M) = \{L(M)_t | t = 0, 1, 2, 3 \dots\}$$

Theorem 1. *For every information system with decision variables $I_t = (U, A, N)$ there exists a Non-homogeneous Rough Finite State Automaton (NRFSA)*

Proof. It can be easily proved by theorem 1 in [1].

Example 4.1. Consider an information system with decision variables, (table 9)

U = {q₀, q₁, q₂, q₃}

Table 9 - Information System with Decision Variables at Time t=1

A/D/U	a_1	a_2	a_3	a_4	a	b
q_0	0.2	0.1	0.8	0.7	0.2	0.5
q_1	0.2	0.1	0.8	0.7	0.2	0.5
q_2	0.1	0.2	0.4	0.6	0.8	0.2
q_3	0.2	0.5	0.6	0.7	0.7	0.4

N = {a, b}

The equivalence classes at time t=1 are,

X₁ = {q₀, q₁}

X₂ = {q₂}

X3 = {q₃}

Let us take $\delta_a = 0.2$ and $\delta_b = 0.2$,

then $\eta_a(1)(q_0) = \{q_0, q_1\}$

$\eta_a(1)(q_1) = \{q_0, q_1\}$

$RS(\eta_a(1)(q_0)) = \{X_1, X_1\}$

$RS(\eta_a(1)(q_1)) = \{X_1, X_1\}$

and for the decision variable 'b',

$\eta_b(1)(q_0) = \{q_0, q_3\}$

$\eta_b(1)(q_1) = \{q_1\}$

$RS(\eta_b(1)(q_0) = \{X_3, X_1 \cup X_3\}$

$RS(\eta_b(1)(q_1) = \{\phi, X_1\}$

Similarly one can calculate the Rough set for the remaining states for both the decision variable a and b (table 10) and by using the above information a Non-homogeneous Rough Finite State Automata M is constructed.

Table 10 - Lower and Upper Approximation of the Decision Variables (Inputs)

$\eta_a(1)/U$	$\eta_a(1)_-$	$\eta_a(1)^-$	$\eta_b(1)_-$	$\eta_b(1)^-$
q_0	X_1	X_1	X_3	$X_1 \cup X_3$
q_1	X_1	X_1	ϕ	X_1
q_2	$X_2 \cup X_3$	$X_2 \cup X_3$	$X_2 \cup X_3$	$X_2 \cup X_3$
q_3	$X_2 \cup X_3$	$X_2 \cup X_3$	$X_2 \cup X_3$	U

$\Sigma = \{a, b\}$ $Q = \{q_0, q_1, q_2, q_3\}$

whereas the Rough transition map δ_1 is defined in (table.9). δ_t is a pair of functions $(\delta(t)_-, \delta(t)^-)$ defined by $\delta(t)_-(q, a) = \eta_a(t)(q)_-$ and $\delta(t)^-(q, a) = \eta_a(t)(q)^-$, similarly for input b it can be defined. Using the transition function $\delta(1)_-$ the Lower Approximation Automata $M(1)_-$ is constructed (fig.1) and by the transition function $\delta(t)^-$ the Upper Approximation Automata$(M(1)^-)$ (fig.2) is constructed.

$R_*(1)$ and $R^*(1)$ for each of the element of U are depicted in the following table (table10 and table 11) $R_*(1)$ (a) = $R_*(1)(b)$ = ϕ implies that, there is no state to which there is a no transition from all the states using the input a and b and by $R^*(1)(a) = R^*(1)(b) = \phi$ implies that possibly none of the states of U are satisfying the input symbols (decision variables) a and b. $R_*(1)$ (q_0) = $R_*(1)(q_1) = \phi$ means that from q_0, there is no states in this neighbourhood for which there is a transition using both the input symbol a and b. Further $R_*(1)(q_2) = R_*(1)(q_3) = X_2 \cup X_3$ which is equal to $\{q_2, q_3\}$ implies that from q_2 using the inputs a and b there is a transition to q_2 and q_3.

Table 11 - $R_*(1)$ and $R^*(1)$ for the Decision Variables (Inputs) of the Information System

R\D	$R_*(1)$	$R^*(1)$
a	ϕ	ϕ
b	ϕ	ϕ

Table 12 - $R_*(1)$ and $R^*(1)$ for all the (Elements of U) States

U\R	q_0	q_1	q_2	q_3
$R_*(1)$	ϕ	ϕ	$X_2 \cup X_3$	$X_2 \cup X_3$
$R^*(1)$	X_1	X_1	$X_2 \cup X_3$	$X_2 \cup X_3$

$R_*(1) (q_0) = R_*(1) (q_1) = \{q_0, q_1\}$ implies that using the inputs a and b possibly there is a transition from q_0 to both of the state q_0 and q_1 and q_1 to q_0 and q_1. Similar interpretations can be given for the states q_2 and q_3.
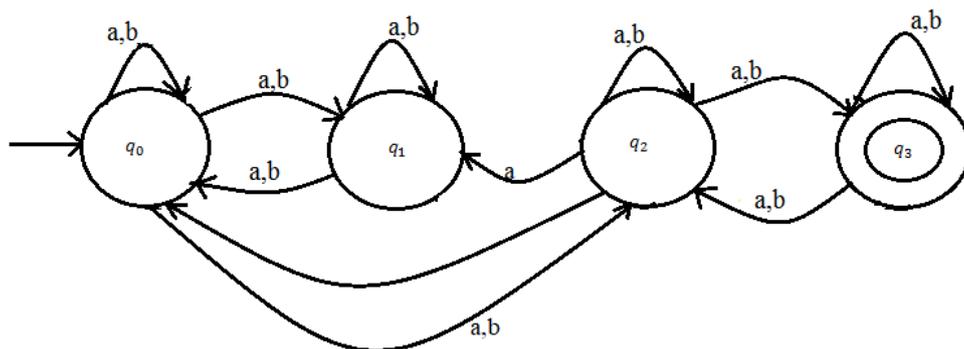
Figure 1 - A Lower Approximation Automata $M(1)_-$
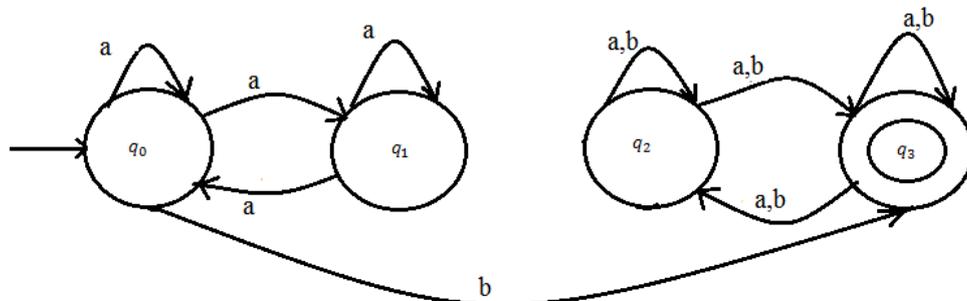
The language for the lower approximation Automata

$$L(M)(1)_- = \{a^*b(a,b)^*\}$$

Figure 2 - The Upper Approximation Automata $M(1)^-$

The language for the upper approximation automata

$$L(M)(1)^- = \{(a,b)^n | n \geq 2\}$$

Consider an information system with decision variables at time t=2 (table 13)

$U = \{q_0, q_1, q_2, q_3\}$

Table 13 - Information System with Decision Variables at Time t=1

A/D/U	a_1	a_2	a_3	a_4	a	b
q_0	0.2	0.1	0.8	0.7	0.2	0.5
q_1	0.2	0.1	0.8	0.7	0.2	0.5
q_2	0.1	0.2	0.4	0.6	0.8	0.2
q_3	0.1	0.2	0.4	0.6	0.7	0.4

N = {a, b}

The equivalence classes at time t=1 are,

$X_1 = \{q_0, q_1\}$

$X_2 = \{q_2, q_3\}$

Let us take $\delta_a = 0.2$ and $\delta_b = 0.2$,

$\Sigma = \{a, b\}$ $Q = \{q_0, q_1, q_2, q_3\}$

Table 14 - Lower and Upper Approximation of the Decision Variables (Inputs)

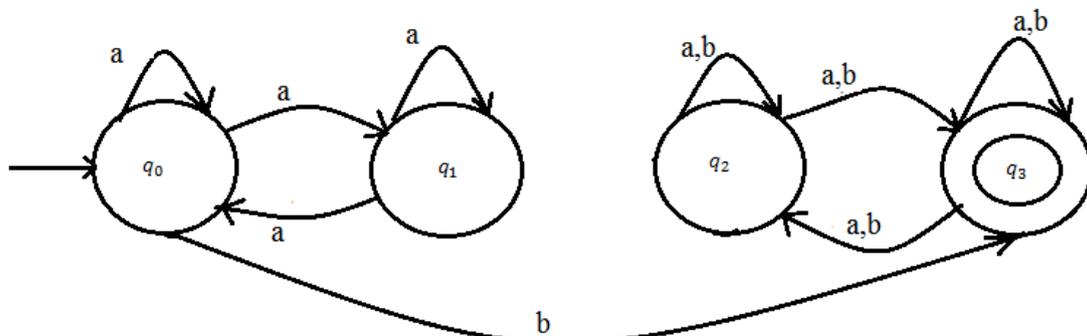| $\eta_a(2)/U$	$\eta_a(2)_-$	$\eta_a(2)^-$	$\eta_b(2)_-$	$\eta_b(2)^-$
q_0	X_1	X_1	ϕ	U
q_1	X_1	X_1	ϕ	X_1
q_2	X_2	X_2	X_2	X_2
q_3	X_2	X_2	X_2	U

Table 15 - $R_*(2)$ and $R^*(2)$ for the Decision Variables (Inputs) of the Information System

R\D	$R_*(2)$	$R^*(2)$
a	ϕ	ϕ
b	ϕ	ϕ

Table 16 - $R_*(2)$ and $R^*(2)$ for all the (Elements of U) States

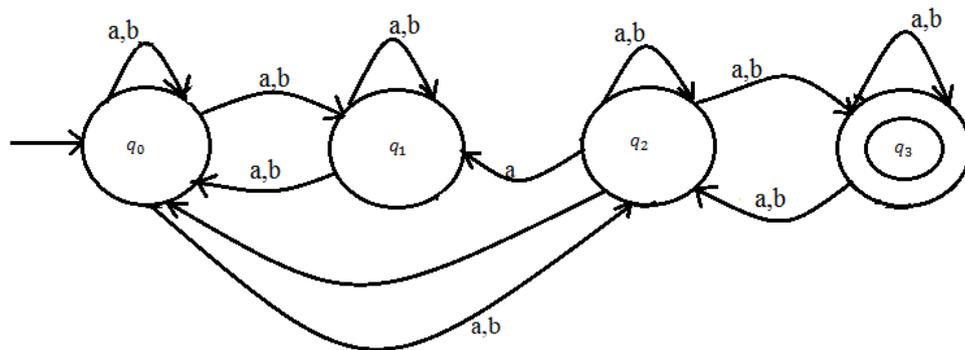| U\R	q_0	q_1	q_2	q_3
$R_*(2)$	ϕ	ϕ	X_2	X_2
$R^*(2)$	X_1	X_1	X_2	X_2

Figure 3 - A Lower Approximation Automata $M(2)_-$

The language for the lower approximation Automata

$$L(M)(2)_- = \{a^*b(a,b)^*\}$$

Figure 4 - The Upper Approximation Automata $M(2)^-$

The language for the upper approximation automata

$$L(M)(2)^- = \{(a,b)^n | n \geq 2\}$$

By the definition 4.2,

$$L(M) = \{L(M)_t | t = 0, 1, 2, 3 \dots\}$$

$$L(M) = \{L(M)(1), L(M)(2)\} = \{a^* b(a,b)^*, (a,b)^n | n \geq 2\}$$

5. Conclusion

In this proposed article we defined non-homogeneous rough finite state automaton (NRF SA) and we calculated the language for rough finite state automaton at each time 't'. All the defined concepts are illustrated with suitable examples. Our future work is to extend this concept to real world problems.

Acknowledgement

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