Analysis of the Conditions of Cotton Packing in Containers with Flexible Casing

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Abstract
The article presents materials on the analysis of the process of cotton compaction in containers with a flexible shell, taking into account vertical pressures and friction forces using approximating functions and nonlinear differential equations.

Key-words: Cotton, Mechanical Seal, Replaceable Containers with Flexible Shell, Algorithms for Numerical Studies, Vertical Pressure Intensities, Friction Forces.

1. Introduction

Processes for studying cotton compaction in bunkers of various types are described in [2,4,6,7,10], in which the conditions of force loading were analyzed and the parameters of a number of designs of mechanical seals were substantiated. It is advisable to further develop these studies in the direction of a more accurate account of the influence of the functions of the intensities of vertical pressures and friction forces on the compaction processes.

In studies [1-4,6,8,9], the expediency of a theoretical analysis of the process of cotton compaction in replaceable containers with a flexible shell, allowing to organize the continuous operation of cotton pickers, is noted. This article presents the materials of studies of the processes of cotton compaction in containers with a flexible shell and substantiates algorithms for numerical studies of these processes.
2. Methods

We can solve this problem taking into account the design schemes (Fig. 1) and using the following assumptions.

We accept the first version of the container with a cross-section, according to Fig. 1, a, most of which along the contour of the shell fits into a circle of radius Ro. To assess the location of points on the surface of the shell, we use the YOZ coordinate system with the placement of the container section in the vertical plane, etc. O from below into the plane of symmetry of the arc of a circle of radius R_o. The top of the container is equipped with a working slot AB, inside of which the cotton mass Gx (t) increasing with time t is supplied. The container has a constant length a, within the 2Ya-a section of the working slot, a uniform supply of cotton mass Gx(t) is ensured. The pressure Po additionally loads the entire cotton mass with a volume of Vk=Fk a (Fk - is the cross-sectional area of the container inside the contour outlined by dots (A,B,C,D,E,F,O). Over time t, an increase in Gx (t) - the density of the cotton mass inside the elastic inextensible shell while maintaining the position of points A and B relative to point O. The model of a compressible cotton mass is characterized by a change in the average density $\gamma_x(t) = \frac{G_x(t)}{V_k}$ while maintaining the volume V_k and the distance H_k. When placed at point Ok at a distance OO_k = Ro of the center of a circle of radius Ro, the contour of the curve AEFODC is described by the equation $Y^2 + (Z - R_o)^2 = R_o^2$, which is inconvenient for subsequent operations of differentiation and integration.

Therefore, we use the approximating function of the curve AEFODC in the form of a polynomial:

$$Y(z) = c_1 z + c_2 z^2 + c_3 z^3, \quad (1)$$

Satisfying the coordinates of the characteristic points V, D and A of this curve, for example $z_B=0.5R_o, \ y_B=0.707R_o$, $z_D=R_o, \ y_D=R_o$, $z_A=1.5R_o, \ y_A=0.707R_o$. 
3. Results

To determine the coefficients $c_1$, $c_2$, $c_3$ of the first version of the container with the specified initial data, we obtain the system of equations:

$$0.707 R_o = 0.5 R_o c_1 + 0.25 R_o^2 c_2 + 0.125 R_o^3 c_3, \quad R_o = R_o c_1 + R_o^2 c_2 + R_o^3 c_3,$$
$$0.707 R_o = 1.5 R_o c_1 + 2.25 R_o^2 c_2 + 3.375 R_o^3 c_3,$$

solving which we get a polynomial:
\[
y_1(z) = 1.712z - \frac{0.482}{R_0} z^2 - \frac{0.23}{R_0^2} z^3 \tag{3}
\]

providing the values \( Y_B = 0.707R_0, \ Y_D = R_0, \ Y_A = 0.708 \), satisfying the accuracy requirements for engineering calculations. With these initial data, the cross-sectional area of the container will be equal to:

\[
F_{k1}(R_0) = 2 \int_0^{1.5R_0} y(z)dz = \left( 1.712z^2 - 0.321 \frac{z^3}{R_0} - 0.115 \frac{z^4}{R_0^2} \right) \left|_0^{1.5R_0} \right. = R_0^2 (3.852 - 1.083 - 0.582) \ F_{k1}(R_0) = 2.87R_0^2
\]

For the second version of the container with the changed: \( z_A = 1.866R_0, \ y_A = 0.5R_0, \) we get a polynomial:

\[
F_{k2}(z) = 1.92z - \frac{1.16}{R_0} z^2 + \frac{0.183}{R_0^2} z^3 \tag{4}
\]

for which: \( F_{k2}(R_0) = (1.92z^2 - 0.733 \frac{z^3}{R_0} - 0.092 - \frac{z^4}{R_0^2}) / 1.332R_0 \)

\[
F_{k2}(R_0) = 2.734 R_0^2 \tag{5}
\]

For the third version of the container with a modified \( z_A = 1.966R_0, \ y_A = 0.259R_0, \) we get a polynomial:

\[
y_3(z) = 1.808z - \frac{0.767}{R_0} z^2 - 0.0409 \frac{z^3}{R_0^2} \tag{6}
\]

for which: \( F_{k3}(R_0) = (1.808z^2 - 0.511 \frac{z^3}{R_0} - 0.0205 z^4) / 1.966R_0 \)

\[
F_{k3}(R_0) = 2.799 R_0^2 \tag{7}
\]

The parameters of the containers differ, according to Fig. 1, and the angles of \( AO_k D: \)

\( 45^0 \) - for the first option, \( 60^0 \) - for the second and \( 75^0 \) - for the third option; for a container of cylindrical section \( F_{k4}(R_0) = 3.14R_0^2 \).

We consider the known function of vertical pressure on the layers of the cotton mass inside the container:

\[
P_z(t,z) = \gamma_z(t)(H_k - z) \tag{8}
\]

and lateral pressure components:

\[
P_y(t,z) = \gamma_y(t)(H_k - z)k_\varphi \tag{9}
\]

with known and constant for the container coefficients of lateral pressure \( k_\varphi \) and friction \( f_\varphi \).

Previous studies revealed that the appearance of the intensity of friction forces on the contact surfaces of the cotton mass with the side walls, the projections of which on the OZ axis turn out to be...
proportional to $\cos^2 \alpha$ ($\alpha$ is the angle of inclination of the tangent of the KK to the vertical plane of the side wall). For the analyzed version of the container according to Fig. 1, a, the angle $\alpha(z)$ at the points of the layers spaced at distances $z$ and $y$ (z) from the point O is a known function of $z$. The calculated values of these angles can be determined based on the equation taking into account (9):

$$\tan \alpha = \frac{dy(z)}{dz} = c_1 + 2c_2z + 3c_3z^2$$

(10)

with the subsequent use of the sin ratio, from which, after transformations, we obtain:

$$\cos^2 \alpha(z) = \frac{1}{\frac{dy(z)}{dz} + 1} = \frac{1}{c_1 + 2c_2z + 3c_3z^2 + 1}.$$  

(11)

These assumptions make it possible to obtain a function of the intensity of friction forces directed along the OZ axis in the direction opposite $P_Z(t, z, y)$:

$$q_T(t, z) = \int_0^z \gamma(t)(H_k - z) \cos^2 dz \ k df_0 = \int_0^z \gamma(t)(H_k - z) dz \ k df_0$$

(12)

The solution to this integral is determined according to [9]:

$$\Delta = 4(c_1 + 1) - 3c_3 - 4c_2^2,$$

(13)

Let us estimate $\Delta$ for containers satisfying polynomials (13):

$$c_1 = 1.712, \ c_2 = -\frac{0.472}{R_0}, \ c_3 = -\frac{0.23}{R_0^2}$$

$$\Delta_1 = -\frac{1}{R_0^2} (7.485 + 0.891) = -\frac{8.376}{R_0^2}$$

(14)

second option: $c_1 = 1.92, \ c_2 = -\frac{1.16}{R_0}, \ c_3 = +\frac{0.183}{R_0^2}$

$$\Delta_2 = 4(1.92 + 1)(+3 \cdot \frac{0.183}{R_0^2}) - 4(-\frac{1.16}{R_0})^2$$

$$\Delta_2 = (6.412 - 5.382) \frac{1}{R_0^2} = +\frac{1.03}{R_0^2},$$

(15)

third option: $c_1 = 1.1808, \ c_2 = -\frac{0.767}{R_0^2}, \ c_3 = -\frac{0.0409}{R_0^2}$

$$\Delta_3 = 4(1.1808 + 1)(-3 \cdot \frac{0.0409}{R_0^2}) - 4(-\frac{0.767}{R_0})^2$$

$$\Delta_3 = -\frac{3.731}{R_0^2}$$

(16)
Now, using integrals №40 and №44 [9,c.92] it is possible to obtain solution (13) for all variants of containers.

According to the calculation scheme (Fig. 1, a), the intensities of friction forces are applied to the points of the curve AEFODC of the flexible shell and, through the forces of connection between the segments and cotton balls, are reduced to the Z axis, along which the intensities of the vertical forces with taking into account (10):

\[ q_Z(t, z) = \int_0^Z P_Z(t, z)dz = \int_0^Z \gamma_X(t)(H_k - z)dz = \gamma_X(t)(H_k z - \frac{z^2}{2}) + q_K. \quad (17) \]

Based on Fig. 1, a, we use the condition for determining \( q_k \) in the form:

\[ P_0 \frac{Y_A}{H_k R_0} \cdot 2Y_A d = \gamma_X(t) \frac{H_k^2}{2} + q_k \text{ откуда } q_k = \left[ \frac{2P_0 Y_A}{H_k R_0} + \gamma_X(t) \frac{H_k^2}{2} \right] \quad (18) \]

and get a clarification:

\[ q_Z(t, z) = \gamma_X(t) \left( H_k Z - \frac{Z^2}{2} - \frac{H_k^2}{2} \right) - \frac{2P_0 Y_A a}{H_k R_0} \quad (19) \]

4. Discussions

We introduce assumptions justifying the functions \( q_z(t, z, y) \), \( q_T(t, z, y) \) and taking into account previously justified similar functions for problems of cotton compaction in bodies:

For the middle section of the container according to Fig. 1, a and point E with \( z=R_0 \) and \( Y=R_0 \) we can take \( q_z(t, z, R_0)=0 \). At the maximum value of this intensity at the point \( O_k - q_z(t, R_0, 0) \); the intensity of friction forces reaches its maximum value at point E - \( q_T(t, R_0, R_0)=q_T(t) \), it can be taken to be zero at point \( O_k - q_T(t, R_0, 0)=0 \).

At point O the maximum is reached \( q_z(t, 0, 0)= q_I(t) \) at \( q_T(t, 0, 0)=0 \); at point B with \( Z = H_k \) and \( Y=0 q_Z(t, H_0, 0) = q_0(t) = \frac{2P_0(t) a Y_A}{H_k} \) is attained in the same layer at points A and C - \( q_Z(t, H_k, Y_a)=0 \).

In the same layer for the \( y=0 \div Y_A \) range, the friction forces can be considered equal to \( q_T(t, H_0, Y_a)=0 \).

These conditions make it possible to obtain approximate functions of the intensities of all forces:
\[ q_c(t, Z, y) = q_1(t) \cos \frac{\pi Z}{2H_k} \cos \frac{\pi Z}{2R_0} + q_0(t) \sin \frac{\pi Z}{2H_k} \cos \frac{\pi Y}{2Y_a} - 2q_f(t) \sin \frac{\pi Z}{R_0} \left(1 - \cos \frac{\pi Y}{2R_0}\right), \quad (20) \]

for which the amplitudes are determined by the formulas:

\[ q_1(t) = \left[ \frac{2P_0Y_a}{H_kR_0} + \gamma_X(t) \frac{H_k^2}{2} \right], \quad q_0(t) = \frac{2P_0Y_a^2a}{H_kR_0} \]

for container variants with \( Z=R_0 \): the first - according to the formula (17), second - by (18), third - by (19).

The model of a compressible mass of cotton inside the container shell is characterized by a variable compression surface \( F_c(Z)=2y(z)a=2a \), for which \( \frac{\partial F(Z)}{\partial Z} = 2a \frac{\partial Y(Z)}{\partial Z} \) is also a given function of \( Z \).

Thus, solving the problem of compacting cotton in a container with a flexible shell requires the use of non-linear equations. We introduce the function \( U_k(t,z,y) \) of elastic deformations of compression of the cotton mass inside the flexible shell and use an equation of the type:

\[ \frac{F_z(Z)\gamma_X(t)}{g} \cdot \frac{\partial^2 U_k}{\partial t^2} + \frac{F_z(Z)}{g} \cdot \frac{\partial \gamma_X(t)}{\partial t} \cdot \frac{\partial U_k}{\partial t} = \frac{\partial F_z(z)}{\partial Z} \cdot \frac{\partial U_k}{\partial Z} - F_k(Z)E \left( \frac{\partial^2 U_k}{\partial Z^2} + \frac{\partial^2 U_k}{\partial Y^2} \right) \cdot q_0(t,z,y), \quad (21) \]

After dividing all the terms of the equation \( F_z(Z)\frac{\gamma_X(t)}{g} \) get:

\[
\frac{\partial^2 U_k}{\partial t^2} + \frac{\partial \gamma_X(t)}{\partial t} \cdot \frac{\partial F_z(z)}{g} \cdot \gamma_X(t) \cdot \frac{\partial U_k}{\partial t} - \frac{\partial F_z(z)}{g} \cdot \frac{\partial U_k}{\partial Z} = \frac{\partial^2 U_k}{\partial Z^2} + \frac{\partial^2 U_k}{\partial Y^2} \cdot q_0(t,z,y) g \]

\[
\cdot \left[ q_1(t) \cos \frac{\pi Z}{2H_k} \cos \frac{\pi Y}{2R_0} + q_2(t) \sin \frac{\pi Z}{2H_k} \cos \frac{\pi Y}{2Y_a} + 2q_f(t) \sin \frac{\pi Z}{R_0} \cos \frac{\pi Y}{2R_0} - 2q_f(t) \sin \frac{\pi Y}{2R_0} \right], \quad (22) \]

This equation requires averaging the coefficients of the functions:

\[
\frac{1}{\tau_y} \int_0^{\tau_y} \frac{\partial \gamma_X(t)}{\partial t} \, dt \times \frac{1}{\tau_y} \int_0^{\tau_y} \frac{\partial F_z(z)}{\partial t} \, dt = \frac{1}{H_k} \int_0^H \frac{\partial F_z(H_k)}{\partial Z} \, dZ = \frac{1}{H_k} \int_0^H \frac{F_z(H_k)}{F_z(o)} = \frac{1}{H_k} \int_0^H y(H_k) \, dZ, \quad (23) \]

However, \( y(0) = 0 \), so the lower integration limit needs to be adjusted. As a container model, we take a truncated cylinder with two symmetric AC type lines (Fig. 1, a) with a reduced \( H_k=2O_zE \). For the third version of the container, such a model will be characterized by \( H_k=1,932R_0 \), and for the second \( H_k=1,732 \). In this case, there will be a decrease in the cross-sectional area of the container for the options: the third by \( 0,325R_0 \) compared to \( F_k=2,799 R_0^2 \), and the second - by \( 0,38 R_0^2 \) compared...
to $F_{kl} = 2.799 R_0^2$. This assumption allows using the function of variable cross-sectional area of cotton mass in a container with lower limits $Z_0 = (0.134+0.034)R_0$ for containers of the second, third options and obtaining equal values $y(z_0) = y(H)$, for which $\frac{1}{H} \ln 1 = 0$.

In the case of using an additional sealing element that introduces portions of cotton through the middle part of the working slot AC (Fig. 1, a) to the depth $H_l$, we determine the mass of the portion:

$$C_{ya} \approx \frac{1}{2} (H \cdot 2Y_A) \alpha \gamma_0 = \alpha \cdot H \cdot Y_A \cdot \gamma_a$$

($(\gamma_a$ - is the average density of the cotton introduced into the working slot), then the pressure amplitude

$$P_a = \frac{H \alpha}{2} Y_A.$$

Let us designate the time intervals for the arrival of the cotton portions $\tau_a$ and the circular frequency $\omega_a = \frac{2 \pi}{\tau_a}$. Instead of the function $q_0 \ell^\beta$, we take a new function of the intensity of vertical pressure forces:

$$q_a(t) = \frac{2 P_a(t) a Y_a}{H_k} = \frac{H \alpha Y_a}{H_k} a Y_A (1 - \cos \omega_a t)$$

and its dependence on $Z$ and $Y$ in the form:

$$q_a(t, z, y) = q_a \sin \frac{\pi z}{2H_k} \cos \frac{\pi y}{2Y_A} (1 - \cos \omega_a t)$$

at $q_a = \frac{H_k Y_A}{H_k} dY_A$. The last function gives rise to a new function of the intensity of friction forces:

$$q_{af}(t, z, y) \approx 2 q_a K_6 f_6 \sin \frac{\pi z}{R_0} (\cos \frac{\pi y}{2R_0} - 1)(1 - \cos \omega_a t)$$

at $q_{af} \approx 2 q_a \cdot Y_A$, due to (Fig. 1, a) geometric dimensions $Y_A$ and $R_0$ of the container.

Ultimately, for this loading case, we obtain an equation of the type:
\[ \frac{\partial^2 U_a}{\partial t^2} - \frac{E_g}{\gamma_{xc}} \left( \frac{\partial^2 U_a}{\partial z^2} + \frac{\partial^2 U_a}{\partial y^2} \right) = \frac{g}{F_Z \gamma_{xc}} \left[ q_i \ell \beta \cos \frac{\pi \ell}{2 R_k} \cos \frac{\pi y}{2 R_0} + q_a (1 - \cos \omega_a t) \right], \quad \text{(28)} \]

We carry out the solution for the remaining terms of equation (27) in the form:

\[ U_a(t, z, y) = U_a(t) \sin \frac{\pi z}{2 H_k} \cos \frac{\pi y}{2 Y_A}, \quad \text{(29)} \]

for which \( U_a(t) \) is found from the solution of the equation:

\[ \frac{d^2 U_a}{dt^2} - \psi_a^2 U_a = \frac{g q_a}{F_Z \gamma_{xc}} (1 - \cos \omega_a t), \quad \text{(30)} \]

at, \( \psi_a^2 = \frac{E_g \pi^2}{4 \gamma_{xc}} \left( \frac{1}{H_k} \right) \), at \( U_a(0) = \frac{d U_a(0)}{dt} = 0 \) we obtain the solution in the form:

\[ U_a(p) = \frac{g q_a \omega_a^2}{F_Z \gamma_{xc}} \frac{1}{(p^2 - \psi_a^2)(p^2 + \omega_a^2)}, \quad \text{(31)} \]

and its original will look like:

\[ U_a(t) = \frac{g q_a \left[ \omega_a^2 (ch \psi_a t - 1) + \psi_a^2 (cos \omega_a t - t) \right]}{F_Z \gamma_{xc} (\omega_a^2 + \psi_a^2) \psi_a^2}, \quad \text{(32)} \]

\[ U_{aT}(t, z, y) = U_{aT} \sin \frac{\pi Z}{R_0} \cos \frac{\pi Y}{2 R_0}, \quad \text{(33)} \]

for which \( U_{aT} \) is found from the equation:

\[ \frac{d^2 U_{aT}}{dt^2} - Y_{aT} = -\frac{2 g q_a Y_A}{R_0 F_Z \gamma_{xc}} (1 - \cos \omega_a t), \quad \text{(34)} \]

at

\[ \psi_{aT}^2 = \frac{E_g \pi^2}{\gamma_{xc}} \cdot \frac{5}{4 R_0^2}, U_{aT}(0) = \frac{d U_{aT}(0)}{dt} = 0, \quad \text{(35)} \]

\[ U_{aT}(t) = \frac{2 K_0 f \varphi g q_A \left[ \omega_a^2 (ch \psi_{aT} t - 1) + \psi_{aT}^2 (cos \omega_a t - 1) \right]}{R_0 F_Z \gamma_{xc} (\omega_a^2 + \psi_{aT}^2) \psi_{aT}^2}, \quad \text{(36)} \]

for which \( U_{ZT} \) is found from the solution of the equation:
\[
\frac{d^2 U_{zt}}{dt^2} - \psi_{zt}^2 U_{zt} = \frac{2K_c \dot{\theta} g q_y g Y}{R_0 F_{zt} \gamma_{xc}} (1 - \cos \omega_a t)
\] (37)

\[
\text{at } \psi_{zt}^2 = \frac{Eg}{\gamma_{xc}} \frac{\pi^2}{R_0^2}; U_{zt}(0) = \frac{dU_{zt}}{dz} = 0
\]

\[
U_{zt}(t) = \frac{2K_c \dot{\theta} g q_y g Y}{R_0 F_{zt} \gamma_{xc}} \left[ \omega_a^2 (\cosh \psi_{zt} t - 1) + \psi_{zt}^2 (\cos \omega_a t - 1) \right]
\] (38)

5. Conclusions

The research results make it possible to implement the following sequence of basic operations for calculating containers with a flexible shell and shape (according to Fig. 1, b).

1. Establishment of the function of the shell curve passing through the points AFODE in the form \(Y(z)\), in the form (1).
2. Establishment of the function of vertical and lateral pressures in the form (8) and (9).
3. Establishment of the \(\cos^2 \alpha(z)\) function in the form of the intensity of friction forces in the form (13).
4. Justification of the functions \(q_c(t,z)\) and \(q_T(t,z,y)\), their representation in the form (18) for the conditions of self-compaction of cotton in a container and from the influence of air pressure.
5. Establishment of functions \(F_z(z)\) of variable section of the compressible cotton mass in the form (19).
6. Solution of equations of the type (21) by evaluating the components of elastic compression deformations and residual deformations from the intensities of external forces and friction on the surfaces of the flexible shell.
7. Analysis of the conditions of pulsed loading of the cotton mass in the working slit due to the action of a special seal, the assessment of the components of deformations of elastic and residual friction forces.
8. Aggregate analysis of the deformation conditions for the middle section (according to the scheme in Fig. 1, a with \(z = R_0\)) or for the section \(z = H_k\) (according to the scheme in Fig. 1, b) with the determination of the maximum attainable density \(\gamma_{xm}\), at which the total elastic displacements from external forces are comparable with the residual displacements from the action of the intensities of the total friction forces.
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